MODERN MATHEMATICAL THOUGHT.

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ONE who, like myself, is not a mathematician in the modern sense naturally feels that some apology is due for accepting the invitation with which this Society has honored me, to address it on a mathematical subject. Possibly an adequate apology may be found in the reflection that one who has not gone deeply into any of the contemporaneous problems of mathematics, but who, as a student, has had a sufficient fondness for the subject to keep himself informed of the general course of thought in it, may be able to take such a general review as is appropriate to the present occasion. I shall therefore ask your consideration of some comparisons between the mode of thinking on mathematical subjects at the present time, and those methods which have come down to us from the past, with a view of pointing out in what direction progress lies, and what is the significance of mathematical investigation at the present day.

Among the miscellaneous reading of my youth was a history of modern Europe, which concluded with a general survey and attempted forecast of progress in arts, science, and literature. So far as I can judge, this work was written about the time of Euler or Lagrange. On the subject of mathematics the writer's conclusion was that fruitful investigation seemed at an end, and that there was little prospect of brilliant discoveries in the future. To us, a century later, this judgment might seem to illustrate the danger of prophesying, and lead us to look upon the author as one who must have been too prone to hasty conclusions. I am not sure that careful analysis would not show the author's view to be less rash than it may now appear. May we not say that in the special direction and along the special lines which mathematical research was following a century ago no very brilliant discoveries have been made? Can we really say that Euler's field of work has been greatly widened since his time? Of the great problems which baffled the skill of the ancient geometers, including the quadrature of the circle, the duplication of the cube, and the trisection of the angle, we have not solved one. Our only advance in treating them has been to show that they are insoluble. To the problem of three bodies we have not added one of the integrals necessary to the complete solution. Our elementary integral calculus is two centuries old. For the general equation of the fifth degree we have only shown that no solution exists. We should, doubtless, solve many of the problems which the Bernoullis and their contemporaries amused themselves by putting to each other, rather better than they did; but, after all, could we get any solution which was beyond their powers? I speak with some diffidence on such a point as this; but it seems to me that progress has been made by going back to elementary principles, and starting out to survey the whole field of mathematical investigation from a higher plane than that on which our predecessors stood, rather than by continuing on the lines which they followed.

We may illustrate this passage to new modes of thought by comparing Euclid's doctrine of ratio and proportion with our own. No one questions the beauty or rigor of the process by which Euclid developed this doctrine in his fifth book, and applied it to the theory of numbers in his seventh book. But can we help pitying our forefathers who had to learn the complex propositions and ponderous demonstrations of the fifth book, all the processes and results of which we could now write on a single sheet of paper? As a mental discipline the study was excellent; but it seems hardly possible that one could have remembered the propositions or the methods of demonstrating them if he had no other knowledge of them than that derived from the work itself. When we carefully examine these propositions, we find that while Euclid recognized the fact that one of two ratios might be greater than, equal to, or less than another, yet he never regarded them as quantities which could be used as operands. From his standpoint a ratio was always a relation, and a relation cannot exist without two terms.

In pointing out this complexity of Euclid's doctrine, I must not be taken to indorse the very loose way in which the doctrine in question is usually treated in our modern textbooks. What we should aim at is to replace Euclid's methods by those which pertain to modern mathematics. At the present time we conceive that a relation between any two concepts of the same kind may always be reduced to a single term by substituting for it an operator whose function it is to change one of these concepts into the other. In the case of the relation between two lines, considered simply as one dimensional quantities, which relation is called a ratio, we regard the ratio as a numerical factor or multiple, which, operating on one line, changes it into the other. For example, that relation which Euclid would have expressed by saying that two lines were to each other as 5 to 2, or that twice one line was equal to five times the other, we should now express by saying that if we multiplied one of the lines by two and one half, we should produce the other. This might seem to be simple difference of words, but it is much more. It is a simplification of ideas; a substitution of one conception for two. Euclid needed two terms to express a relation; we need but one.

But this is not the only simplification. A peculiarity of our modern mathematics is that operators themselves are regarded as independent objects of reasoning; susceptible of becoming operands, without specification of their particular qualities as operators. Thus, instead of considering the ratio which I have just mentioned as an operation of multiplying a line by two and one half, we finally reduce it to the simple quantity two and one half, which we may conceive to remain inert until we bring it into activity as a multiplier. It thus assumes a concrete form, capable of being carried about in thought, and operated upon as if it were a single thing.

This example may afford us a starting-point for a farther illustration of the way in which we have broadened the conceptions which lie at the basis of mathematical thought. Let us reflect upon the relation between a straight line going out from a certain point, and another line of equal length going out from the same point at right angles to the first. Had this relation been presented to Euclid as a subject of study, he would probably have replied that though much simpler than those he was studying, he could see nothing fruitful in it, and would have drawn no conclusions from it. But if we trace up the thought we shall find a wide field before us, embracing the first conception of groups, and with it an important part of our modern mathematics. In accordance with the principle already set forth, we replace the relation between these two lines by an operator which will change the first into the second. We define this operator by saying that its function is to turn a line through a right angle in a fixed plane containing the line. This definition permits of the operator in question being applied to any line in the plane. Then let us apply it twice in succession to the same line. The result will be a line pointing in the opposite direction from the original one. A third operation will bring it again to a right angle on the opposite side from the second position; and a fourth will restore the line to its original position, the result being to carry it through a complete circle. If we now consider the operations which would have been equivalent to these one, two, three, and four revolutions through a right angle as four separate operators, we see that their results will be either to leave the line in its original position, or to move it into one of three definite positions. If we then repeat any of these four operations as often as we please or in any order we please, we shall only bring the line to one of the four positions in question. We thus have a group of the fourth order, possessing the property that the repetition of any two operations of the group is equivalent to some single operation of it.

I scarcely need call attention to the familiar homology between these operations and successive multiplications by the imaginary unit. This last concept, considered as a multiplier, has the same properties as our rotating operator. Repeated twice, it changes the sign or direction of the quantity on which it operates; repeated four times, it restores it to its original value.

We have here a very simple illustration of a law of thought the application of which forms the basis of an important part of modern mathematical research. We may call it the law of homology. I am not sure of my ability to define it rigorously, but I think we may express it in some such form as this: If we have two sets of concepts, say A and B, such that to every concept of the one set shall correspond a concept of the other, and to every relation between any two of one set a corresponding relation between the corresponding two of the other, then all language, reasoning, and conclusions as to the one set may be applied to the other set. We may, of course, extend the law to a correspondence between things or concepts, and symbols or other forms of language.

This law is, I think, more universal than might at first sight appear. Not only the progress but the very existence of our race depends upon that co-ordination between our mental processes and the processes of the external universe, which has gradually been brought about by the attrition between man and nature through unnumbered generations. A man is perfect, powerful, and effective in proportion as his thoughts of nature coincide with the processes of nature herself; each process of nature having its image in his thought, and *vice versa*. Now language consists in a co-ordination between words and conceptions. Thus we pass from nature to what corresponds to it in thought, and from thought to what corresponds to it in language, and thus bring about a correspondence between language and nature.

Modern scientific research affords many examples of the application of this law, which would be very marvellous if they were not so familiar. We are so accustomed to the prediction of an eclipse that we see no philosophy in it. And yet might not a very intellectual being from another sphere see something wonderful in the fact that by a process of making symbols with pen and ink upon sheets of paper, and combining them according to certain simple rules, it is possible to predict with unerring certainty that the shadow of the moon, on a given day and at a given hour and minute, will pass over a certain place on the earth's surface? Surely the being might ask with surprise how such a result could be attained. Our reply would be simply this: There is a one-to-one correspondence between the symbols which the mathematician makes on his paper, and the laws of motion of the heavenly bodies. His symbols embody the methods of nature itself.

The introduction and application of homologies such as I have pointed out have, perhaps, their greatest value as thought-savers. In the field of mathematical thought they bear some resemblance to labor-saving machines in the field of economics. They enable the results of ratiocination to be reached without going through the process of reasoning in the particular case. Much that I have said illustrates this use of the method, but there is yet another case which has been so fruitful as to be worthy of special mention: I mean the general theory of functions of an imaginary variable. We may regard such functions as being in reality representative of a pair of functions of a certain class involving a pair of real variables; but the difficulty of conceiving the various ways in which the two variables might be related, and the results of the changes which they might go through, in such a way as to clearly follow out all possible results, would have rendered their direct study impossible.

But when Gauss and Cauchy conceived the happy idea of representing two such variables, the real and the imaginary one, by the rectangular co-ordinates of a point in a plane, those relations which before taxed the powers of conception became comparatively simple. Considered as a magnitude, the complex variable, or the sum of a real quantity and a purely imaginary one, the latter being considered as one measured in imaginary units, was represented by the length and position of a straight line drawn from an origin of co-ordinates to the point whose co-ordinates were represented by the values of the variable. Such a line, when both length and direction are considered, is now familiarly known as a vector. The conception of the vector would, however, in many cases be laborious. But the vector is completely determined by its terminal point; to every vector corresponds one and only one terminal point, and to every terminal point one and only one vector. Hence we may make abstraction of the vector entirely, and in thought attend only to the terminal point. Since for every pair of values we assign to our original variables there is one point, and only one, we may in thought make abstraction of both of these variables, and of the vectors which they represent, and consider only the point whose co-ordinates they are. Thus the continuous variation of the two quantities, how complex soever it may be, is represented by a motion of the point. Now such a motion is very easy to conceive. We may consider it as performing a number of revolutions around some fixed position without the slightest difficulty, whereas to conceive the corresponding variations in the algebraic variables themselves would need considerable mental effort. Thus, and thus alone, has the beautiful theory, first largely developed by Cauchy, and afterward continued by Riemann, been brought to its present state of perfection.

Another example of the principle in question, where the two objections of reasoning are so nearly of a kind that no thought is saved, is afforded by the principle of duality in projective geometry. Here a one-to-one correspondence is established between the mutual relations of points and lines, with the result that in demonstrating any proposition relating to these concepts we at the same time demonstrate a correlative proposition formed from the original one by simply interchanging the words "point" and "line."

The subjects of which I have heretofore spoken belong conjointly to algebra and geometry. Indeed, one of the great results of bringing homologous interpretation into modern mathematics has been to unify the treatment of algebra and geometry, and almost fuse them into a single science. To a large class of theorems of algebra belong corresponding theorems of geometry, each of one class proving one of the other class. Thus the two sciences become mutually helpful. In geometry we have a visible representation of algebraic theorems; by algebraic operations we reach geometrical conclusions which it might be much more difficult to reach by direct reasoning. A remarkable example is afforded by the geometrical application of the theory of invariants. These are perhaps the last kind of algebraic conclusions which the student, when they are first presented to his attention, would conceive to have a geometrical application. Yet a very little study suffices to establish a complete homology between them and the distribution of points upon a straight line.

This use of homologies does not mark the only line by which we have advanced beyond our predecessors. Progress has been possible only by emancipating ourselves from certain of the conceptions of ancient geometry which are still uppermost in all our elementary teaching. The illustration I have already given is here much to the point. The expression of a relation between two straight lines by the multiplier which would change one into the other is now familiar to every schoolboy, and the relation itself was familiar to Euclid. But the yet simpler relation of a line to another of equal length standing at right angles to it, and the corresponding operator which will change one into the other, was never thought of by Euclid, and is unfamiliar in our schools.

Why is this? It seems to me that it grows out of the ancestral idea that mathematics concerns itself with measurement, and that the object of measurement is to express all magnitudes in one-dimensional measure. So completely has this idea directed language, that we still extend the use of the word "equal" to all cases of this particular kind of linear equality: we say that a circle is equal to the rectangle contained by its radius and half its circumference. We have therefore been obliged to invent the word "congruent" for absolute equality in all points, or to qualify the adjective "equal" by "identical," saying "identically equal." There is of course no objection to the comparison of magnitudes in this way by reference to one dimensional measures, or by presupposing that the change which one magnitude must undergo in order to be transformed into the other is to be expressed by a single parameter, but changes involving two or any number of parameters, are just as important as those involving one, and the attempt to express all metric relations by referring them to a single parameter has placed such restrictions on thought that it seems to me appropriate to apply the term emancipation to our act in freeing ourselves from them. With us mathematics is no longer the science of quantity. But even if we consider that the ultimate object of mathematics is relations between quantities, we have reaped a rich reward by the emancipation, for we are enabled by the use of our broader ideas to reach new conclusions as to metric relations.

I trust that my discourse will not be found too academic if I continue with some further illustrations of the homologies of groups. Returning to the relations between lines, let us suppose that instead of taking two lines at right angles to each other, we consider two which form an angle of 10 degrees. As already remarked, this relation is homologous with an operator which will turn a single line through that angle. If we continually repeat this operation, we shall bring the line into 35 different positions, the 36th position being identical with the original one. Thus we should have 36 positions in all, expressed by that number of lines radiating from a single centre, and making angles of 10 degrees with each other. Now let us imagine 36 operators whose function it is to turn a line, no matter what, successively through an arc of 10 degrees, 20 degrees, 30 degrees, etc., up to 360 degrees, the last being equivalent to an operator which simply does nothing. These 36 operators will form a group which we know to be strictly homologous with multiplication by the 36 expressions

$$e^{i\phi},\ e^{2i\phi},\ e^{3i\phi}\ \ldots\ e^{36i\phi} = e^0 = 1,$$

where ϕ is the arc of 10° in circular measure.

So far we have only considered operations formed by the continual repetition of a single one; in the language of the subject, all our groups are constructed from powers of a single operator. Now let us extend our process by substituting a cube for our straight line. Through this cube we have an axis parallel to four of its plane sides. By rotating the cube through any multiples of 90 degrees around this axis we effect an interchange of position between four of its sides. This process of interchanging is homologous with rotation through 90 degrees, being in fact equivalent to it, and therefore it is also homologous with multiplication by the imaginary unit. But there is also another homology. Let us designate the four sides of the cube parallel to the axis of rotation as A, B, C, D. Then our group of rotations will be homologous with the powers of a cyclic substitution between the four letters A, B, C, D.

Let us next introduce a new operator, namely, rotation around an axis at right angles to the first one, but always through an arc of 90 degrees. This introduces a new element into the problem, and enables us to change the cube from any one position to any other position, that is, to effect any interchange among the sides which would be consistent with their remaining sides of the same cube. Here we have a series of rotations which, in the case of the cube, are homologous with certain linear transformations which have been developed by Klein in his very beautiful book on the Icosahedron.

But it is also obvious that in introducing these rotations we are practically operating with quaternions, the operator being a unit vector. Thus we have a homology between certain forms of quaternion multiplications and linear transformations involving the imaginary unit. Moreover, since these rotations are also homologous with substitutions, performed on six symbols representing the six sides of the cube, it follows that there is also a homology between certain groups of substitutions and certain linear transformations involving two quantities, a numerator and a denominator, and quaternion multiplication by unit vectors.

I have taken a cube as the simplest illustration. Evidently we can construct a great number of groups of substitutions of the same sort between the sides of any regular solid, as Klein has done in the work I have already cited. The relation between the linear substitutions thus found and the solution of corresponding algebraic equations forms one of the most beautiful branches of our modern mathematics.

The idea of groups of operations, as I have tried to develop it, has in recent years been so extended as to cover a large part of the fields of algebra and geometry. Among the leaders in this extension has been Sophus Lie. Considered from the algebraic point of view, his idea in its simplest form may be expressed thus: We have a certain quantity, say x. We have also an operation of any sort which we may perform upon this quantity. Let this operation depend on a certain quantity, a, which necessarily enters into it. As one of the simplest possible examples we may consider the operation to be that of adding a to x. As the quantity a may take an infinity of values, it follows that there will be an infinity of operations all belonging to one class, which operations will be distinguished by the particular value of a in each case. We thus operate on x with one of these operators, and get a certain result, say x'. We operate on x' with a second operator, of the same class, and get a second result, say x''. If whatever operators we choose from the class, the result x'' could have been obtained from the original quantity x by some operation of the class, then these operations are such that the product of any two is equivalent to the performance of some one of them. Thus, by repeating them forever, we could get no results except such as could be obtained by some one operator. To illustrate by our simple example: if our operation consists in the addition of an arbitrary quantity to x, then we change x into x' by adding a certain quantity a, and x' into x'' by adding a second quantity b. The result of these two additions is the same as if we had added in the first place the quantity $a+b$. It needs hardly be said that the multiplication by x of any quantity would be another example of the same kind. The performance of any number of successive multiplications on a quantity is always equal to a single multiplication by the product of all the factors of the separate multiplications.

These operations are not confined to single quantities. We may consider the operation to be performed upon a system of quantities, which are thus transformed into an equal number of different quantities, each of these new quantities corresponding to one of the first system. If a repetition of the operation upon the second system of quantities gives rise to a third system, which could have been formed from the first system by an operation of the same class, then all these possible operations form a group.

The idea of such systems of operations is by no means new. It has always been obvious, since the general theory of algebraic operations has been studied, that any combination of the operations of addition, multiplication, and division could always be reduced to a system in which there would be only a single operation of division necessary —just as in arithmetic a complex fraction, no matter what the order of complexity of its terms, can always be reduced to a single simple fraction, that is, to a ratio of two integers, but cannot, in general, be reduced to an integer. Abel made use of this theorem in his celebrated memoir on the impossibility of solving the general equation of the fifth degree.

Another field of mathematical thought, quite distinct from that at which we have just glanced, may be called the fairy-land of geometry. To make a mathematician, we must have a higher development of his special power than falls to the lot of other men. When he enters fairyland he must, to do himself justice, take wings which will carry him far above the flights, and even above the sight, of ordinary mortals. To the most imaginative of the latter, a being enclosed in a sphere, the surface of which was absolutely impenetrable, would be so securely imprisoned that not even a spirit could escape except by being so ethereal that it could pass through the substance of the sphere. But the mathematical spirit, in four-dimensional space, could step out without even touching any part of the globe. Taking his stand at a short distance from the earth, he could with his telescope scan every particle of it; from centre to surface, without any necessity that the light should pass through any part of the substance of the earth. If a practised gymnast, he could turn a somersault and come down right side left, just as he looks to our eyes when seen by reflection in a mirror, and that without suffering any distortion or injury whatever. A straight line, or a line which to all our examination would appear straight, if followed far enough might return into itself. Space itself may have a boundary, or, rather, there may be only a certain quantity of it; go on forever, and we would find ourselves always coming back to the starting-point. These results, too, are not reached by a facetious use of words, but by rigorous geometrical demonstration.

The considerations which lead to the study of these forms of space are so simple that they can be traced without difficulty. When the youth begins the study of plane geometry, his attention is devoted entirely to figures drawn upon a plane. For him space has only two dimensions. To a given point on a straight line only one perpendicular can be drawn. By moving a line of any sort in the plane he can describe a surface, but a solid is wholly without his field. He cannot draw a line from the outside to the inside of a circle without intersecting it. On a given base only two triangles with given sides can be erected, one being on one side of the base, the other on the other. When he reaches solid geometry his conceptions are greatly extended. He can draw any number of perpendiculars to the same point of a straight line. If he has two straight lines perpendicular to each other, he can draw a third straight line which shall be perpendicular to both. A plane surface is not confined to its own plane, but can be moved up and down in such a way as to describe a solid. The characteristic of this motion is that it constantly carries every part of the plane to a position which no part occupied before.

Now it is a fundamental principle of pure science that the liberty of making hypotheses is unlimited. It is not necessary that we shall prove the hypothesis to be a reality before we are allowed to make it. It is legitimate to anticipate all the possibilities. It is, therefore, a perfectly legitimate exercise of thought to imagine what would result if we should not stop at three dimensions in geometry, but construct one for space having four. As the boy, at a certain stage in his studies, passes from two to three dimensions, so may the mathematician pass from three to four dimensions. He does indeed meet with the obstacle that he cannot draw figures in four dimensions, and his faculties are so limited that he cannot construct in his own mind an image of things as they would look in space of four dimensions. But this need not prevent his reasoning on the subject, and one of the most obvious conclusions he would reach is this: As in space of two dimensions one line can be drawn perpendicular to another at a given point, and by adding another dimension to space a third line can be drawn perpendicular to these two; so in a fourth dimension we can draw a line which shall be perpendicular to all three. True, we cannot imagine how the line would look, or where it would be placed, but this is merely because of the limitations of our faculties. As a surface describes a solid by continually leaving the space in which it lies at the moment, so a four dimensional solid will be generated by a three-dimensional one by a continuous motion which shall constantly be directed outside of this three-dimensional space in which our universe appears to exist. As the man confined in a circle can evade it by stepping over it, so the mathematician, if placed inside a sphere in four-dimensional space, would simply step over it as easily as we should over a circle drawn on the floor. Add a fourth dimension to space, and there is room for an indefinite number of universes, all alongside of each other, as there is for an indefinite number of sheets of paper when we pile them upon each other.

From the point of view of physical science, the question whether the actuality of a fourth dimension can be considered admissible is a very interesting one. All we can say is that, so far as observation goes, all legitimate conclusions seem to be against it. No induction of physical science is more universal or complete than that three conditions fix the position of a point. The phenomena of light show that no vibrations go outside of three-dimensional space, even in the luminiferous ether. If there is another universe, or a great number of other universes, outside of our own, we can only say that we have no evidence of their exerting any action upon our own. True, those who are fond of explaining anomalous occurrences, by the action of beings that we otherwise know nothing about, have here a very easy field for their imagination. The question of the sufficiency of the laws of nature to account for all phenomena is, however, too wide a one to be discussed at present.

As illustrating the limitation of our faculties in this direction, it is remarkable that we are unable to conceive of a space of two dimensions otherwise than as contained in one of three. A mere plane, with nothing on each side of it, is to us inconceivable. We are thus compelled, so far as our conceptions go, to accept three dimensions, and no more. We have in this a legitimate result of the universal experience through all generations being that of a triply extended space.

Intimately associated with this is the concept of what is sometimes called curved space. I confess that I do not like this expression, as I do not see how space itself can be regarded as curved. Geometry is not the science of space, but the science of figures in space, possessing the properties of extension and mobility which we find to be common to all material bodies. The question raised here is a very old one, and in a general way its history is familiar.

Mathematicians have often attempted to construct geometry without the use of what is commonly called the ninth axiom of Euclid, which seems to me best expressed by saying that in a plane only one line can be drawn which shall be parallel to another line in the plane in the sense of never meeting it in either direction. Yet every attempt to construct an elementary geometry without this axiom has been proved to involve a fallacy. This consideration led Lobatchewsky, and independently of him, I believe, Gauss, to inquire whether a geometry might not be constructed in which this axiom did not hold; in which, in fact, it was possible that if we had two parallel lines in a plane, one of them might turn through a very minute amount without thereby meeting the other line in either direction. The possibility of this was soon shown, and a system of geometry was thus constructed in which the sum of the angles of a plane triangle might be less than two right angles.

Afterward the opposite hypothesis was also introduced. It was found that, given two parallel lines in a plane, it might be supposed that they would ultimately meet in both directions. This hypothesis might even be made without there being more than one point of intersection, each straight line having a definite end and returning into itself. The geometry arising from these two hypotheses has been reduced to a rigorous system by Klein.

To guess the future of mathematical science would be a rash attempt. If made it might seem that, in view of the extraordinary works of the human intellect which mark our age, the safest course would be to predict great discoveries in this and all other branches of science. The question is sometimes asked whether a mathematical method may not yet be invented which shall be as great an advance on the infinitesimal calculus as the latter was on the methods of Euclid and Diophantus.

So far as solving problems which now confront us is concerned, I am not sure that the safest course would not be to answer such questions in the negative. Is it not true in physics as in mathematics that great discoveries have been made on unexpected lines, and that the problems which perplexed our ancestors now baffle our own efforts? We must also remember that the discovery of what could not be done has been an important element in progress. We are met at every step by the iron law of the conservation of energy: in every direction we see the limits of the possible. The mathematics of the twenty-first century may be very different from our own; perhaps the schoolboy will begin algebra with the theory of substitution-groups, as he might now but for inherited habits. But it does not follow that our posterity will solve many problems which we have attacked in vain, or invent an algorithm more powerful than the calculus.

RECENT RESEARCHES IN ELECTRICITY AND MAGNETISM.

Notes on Recent Researches in Electricity and Magnetism. Intended as a sequel to Professor Clerk-Maxwell's "Treatise on Electricity and Magnetism." By J. J. Thomson, M.A., F.R.S. Oxford, Clarendon Press, 1893. 8vo. pp. 586.

POINCARÉ remarks in his "Électricité et Optique": "La première fois qu'un lecteur français ouvre le livre de Maxwell, un sentiment de malaise, et souvent même de défiance se mêle d'abord à son admiration." And again he says: "Le savant anglais ne cherche pas à construire un édifice unique, définitif et bien ordonné, il semble plutôt qu'il élève un grand nombre de constructions provisoires et indépendantes, entre lesquelles les communications sont difficiles et quelquefois impossibles." The disconnected way in which Maxwell takes up one hypothesis after another, and then leaves his readers to select for themselves, is naturally abhorrent to a great