

theorems represented by Sturm's famous paper in the first volume of *Liouville's Journal*. When we consider the great importance of these questions both theoretically and in physical applications it is hard to justify their omission in what claims to be a Handbook of the whole theory of linear differential equations. This omission may, perhaps, be in part made good in the second volume.

Although Professor Schlesinger's treatise fails to meet some of the demands which it seems to us may fairly be made of a Handbook, it is certain, owing to the great amount of information which it contains in accessible form, to fill an important place in every mathematical library.

MAXIME BÔCHER.

HARVARD UNIVERSITY,
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TABLE OF THE FIRST FORTY ROOTS OF THE
BESSEL EQUATION $J_0(x) = 0$ WITH THE COR-
RESPONDING VALUES OF $J_1(x)$.

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BY R. W. WILLSON AND B. O. PEIRCE.

THE first ten values of x for which Bessel's function of the zeroth order, $J_0(x)$, vanishes have been given to ten places of decimals by Meissel.* The next thirty roots of the equation, $J_0(x) = 0$ and the values of $J_1(x)$ corresponding to these forty roots have been computed by us by means of Vega's ten place table of logarithms† except in the few cases where a greater number of places was necessary, and for these we have had recourse to Thoman's tables.‡ All the values have been checked by duplicate computation and the first four values of $J_1(x)$ by comparison with Meissel's tables.

* *Math. Abhandlungen der K. Akad. der Wissenschaften zu Berlin*, 1888.

† *Thesaurus Logarithmorum Completus*, Lipsiae, 1794.

‡ *Tables de Logarithmes à 27 Décimales pour les Calculs de Précision*, Paris, 1867.

The p^{th} root in order of magnitude of the equation $J_0(x) = 0$ is denoted by x_p .

p	x_p	$\text{Log } x_p$	$J_1(x_p)$	$\text{Log}(\pm J_1(x_p))$
1	2.4048255577	0.3810835788	+0.51914750	9.7152908
2	5.5200781103	0.7419452231	-0.34026481	9.5318170
3	8.6537279129	0.9372032361	+0.27145230	9.4336935
4	11.7915344391	1.0715703238	-0.23245933	9.3663479
5	14.9309177086	1.1740865018	+0.20654642	9.3150177
6	18.0710639679	1.2569337232	-0.18772880	9.2733509
7	21.2116366299	1.3265741787	+0.17326589	9.2387131
8	24.3524715308	1.3865430443	-0.16170155	9.2087142
9	27.4934791320	1.4392297006	+0.15218121	9.1823610
10	30.6346064684	1.4862123057	-0.14416598	9.1588628
11	33.7758202136	1.5286059043	+0.13729694	9.1376609
12	36.9170983537	1.5672275586	-0.13132463	9.1183462
13	40.0584257646	1.6026938781	+0.12606950	9.1006100
14	43.1997917132	1.6354816528	-0.12139863	9.0842138
15	46.3411883717	1.6659671666	+0.11721120	9.0689691
16	49.4826098974	1.6944525978	-0.11342918	9.0547248
17	52.6240518411	1.7211842839	+0.10999114 ₅	9.0413577
18	55.7655107550	1.7463656842	-0.10684789	9.0287659
19	58.9069839261	1.7701667872	+0.10395937	9.0168645
20	62.0484691902	1.7927310714	-0.10129350	9.00555816

The p^{th} root in order of magnitude of the equation $J_0(x) = 0$ is denoted by x_p .

p	x_p	$\text{Log } x_p$	$J_1(x_p)$	$\text{Log}(\pm J_1(x_p))$
21	65.1899648002	1.8141807465	+0.09882255	8.9948561
22	68.3314693299	1.8346207594	-0.09652404	8.9846355
23	71.4729816036	1.8541418997	+0.09437879	8.9748744
24	74.6145006437	1.8728232368	-0.09237051	8.9655333
25	77.7560256304	1.8907340543	+0.09048519	8.9565775
26	80.8975558711	1.9079354006	-0.08871080	8.9479765
27	84.0390907769	1.9244813451	+0.08703686	8.9397032
28	87.1806298436	1.9404200023	-0.08545424	8.9317336
29	90.3221726372	1.9557943757	+0.08395493	8.9240462
30	93.4637187819	1.9706430570	-0.08253186	8.9166216
31	96.6052679510	1.9850008094	+0.08117879	8.9094426
32	99.7468198587	1.9988990584	-0.07989015	8.9024933
33	102.8883742542	2.0123663047	+0.07866100	8.8957595
34	106.0299309165	2.0254284784	-0.07748689	8.8892282
35	109.1714896498	2.0381625681	+0.07635913	8.8828610
36	112.3130502805	2.0504302219	-0.07528823	8.8767271
37	115.4546126537	2.0624112882	+0.07425684	8.8707365
38	118.5961766309	2.0740706879	-0.07326670	8.8649067
39	121.7377420880	2.0854252422	+0.07231515	8.8592293
40	124.8793089132	2.0964904866	-0.07139973	8.8536966