NEWTON'S THEORY OF KINETICS.

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Our knowledge of kinetics is primarily derived from experiments on terrestrial-motions. Galileo made the first important step by his study of the motions of falling bodies and projectiles relative to the earth. He eliminated the effects of friction and of the resistance of the air, and discovered the laws, which are approximately correct, of the uniform acceleration of a body falling or sliding on an inclined plane, and of the parabolic motion of a projectile. His investigations do not touch the question of the introduction of any correction involving the earth's rotation.

The earliest recorded suggestion of such a correction being needed is due to Newton. Starting with the view, which he regarded as generally accepted, that an isolated particle would move uniformly relatively to some base, he was doubtless convinced by his study of the planetary motions that this base, if it existed, was not the earth. In a letter to Hooke, dated November 28, 1679,* he points out that the question whether the earth has a diurnal motion, relative to this supposed base, could be put to the test by observing whether the path of a body, falling freely from rest, diverges from the vertical line, indicated by a plumb line. He describes in detail the way in which the experiment might be carried out, his expectation being that there would be a deflection to the east, "quite contrary," he says, "to the opinion of the vulgar who think that, if the earth moved, heavy bodies in falling would be outrun by its parts and fall on the west side of the perpendicular." Hooke made the experiment, but it is doubtful whether the results which he obtained were of much value. He thought that a deviation due to the earth's rotation should be more to the south than to the east, and he professed to find a considerable southeasterly deviation. Moreover in one case, in which a ball was dropped into a box full of clay, marked with cross lines, the height from which it fell was only 27 feet, for which the deviation should have been only about one hundredth of an inch. What weight, if any, Newton attached to Hooke's results does not appear, but he adhered to the view that a base of reference should be adopted with a diurnal rotation, and probably also an acceleration, relative to the earth, and set himself to modify the existing

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* *An Essay on Newton's Principia*, by W. W. Rouse Ball, p. 142.

theory in such a way as to fit it for universal application. The statement which he produced is contained in the two introductory chapters of the Principia, the titles of which are "Definitiones" and "Axiomata sive leges motus."

It is important to notice how difficult the task was which Newton undertook. He saw the need for a new base of reference, and that this base could only be defined by tests supplied by the theory. He saw also that force (or "impressed force" as he calls it) must be measured in terms of motion. Thus the various points to be dealt with interacted among themselves in an embarrassing way. Moreover the measurement of quantity of motion required generalizing. Under these conditions he appears not to have attempted to devise a statement in strict logical sequence. He seems to have contented himself with describing his new point of view in the language which he had accustomed himself to use, modifying the existing theory, such as it was, instead of wholly recasting it. Thus the relations between force and quantity of motion or mass and base of reference are allowed to appear incidentally, instead of being set out in an orderly sequence. His avoidance of blunders, in spite of the confused arrangement, affords strong evidence of the soundness of his views, and justifies us in adopting a favorable interpretation of passages in which the explanations given are incomplete. In the following examination of his statement this is the light in which it is regarded.

The point which it is natural to consider first in the measurement of time is a physical quantity. Newton discusses this question in the Scholium appended to the definitions. He gives no sanction to the view, which many writers have adopted, that the measure of time is defined by the first law of motion. On the contrary he appears to regard the measurement of time as a matter independent of his theory. He states that there is a standard time, which he calls "absolute time," and chiefly concerns himself with insisting upon the distinction between this and what he calls "relative time," the name which he seems to apply to time as measured by any particular physical contrivance. A year, a month and a day are the times which elapse between the occurrences of certain astronomical events, and an hour is to be regarded as a certain fraction of a day; thus all these measures of time are referred to as "relative." Absolute time, he says, flows equally without relation to anything apart from itself; and so Professor Mach*

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* *The Science of Mechanics* (American edition), p. 224.

has accused him of introducing an idle metaphysical conception of no practical value. Newton, no doubt, to some extent lays himself open to such an accusation; but the point which he insists upon is perfectly correct. The distinction between time as measured by the earth's rotation, or any clock, and the ultimate standard to be used in physics is a real and important one. The question is what indication Newton gives of the tests by means of which this standard may be referred to. Such an indication occurs in the last sentence of his discussion of the subject. He has been speaking of the "equation of time" as the correction to be applied to time as measured by solar days to give absolute time, and he says that the necessity of this correction is shown both by experiments with the pendulum clock, and by the eclipses of the satellites of Jupiter. These words prove that he did not regard his standard as beyond the pale of physical test, and give, so far as they go, a correct indication of the nature of the tests to be applied. The ultimate test of equal times appears, in fact, to be based upon the comparison of the results of various examples of what may be called repetition methods of measurement. The numerical measure of a period of time might be defined as being the number of times a given physical operation would take place in the course of it, if repeated under identical circumstances, without any interval between the repetitions. This definition cannot be directly applied, since no two events can happen with absolutely identical surroundings; thus the conception of an exact ratio between two periods of time is derived from the comparison of the results given by a variety of methods which approximately fulfil the conditions laid down in the definition, results which are on the whole consistent. There is no reason to suppose that the mental comparison of two periods of time, based upon a succession of thoughts, is either inconsistent with the physical test or independent of it. The rotation of the earth, relative to the fixed stars, is referred to as affording a practical standard because, after comparison with other repetition methods, it is found that the average results agree, but that the performance of the earth is better than that of any clock.

The Scholium in which the measurement of time is discussed deals also with the question of the base of reference to be used for space and motion. In fact, time and space are dealt with together, the same language being applied to both. And as, in the case of time, the author is chiefly concerned with distinguishing between the measures of time by particular contrivances and the ideal measure which is the standard for scientific purposes, so, in the case of space and motion, he is chiefly concerned with distinguishing between motion relative to any particular body and what he chooses to call "absolute motion." The distinction is one of equal importance in the theory. The weak point in Newton's treatment of it is that he insists too much upon the fundamental character of his base. Instead of saying that the theory establishes a certain base of reference, in terms of which the relative motions of bodies are capable of peculiarly simple expression, he seems to assume the existence of a fundamental base, as if it were a thing already known, and then turns to the question of the tests by which and the extent to which it can be identified. This inversion in logical arrangement makes no difference to the scientific result, but it possibly points to a certain amount of confusion of ideas as to the exact limits of the ground covered by the theory. The confusion, if it can be so called, shows itself in another point of arrangement; for in the Scholium "absolute space" is referred to as a definite framework relative to which absolute position can be assigned, and it is not till the next chapter (corollary V), that the author completes the statement by making the necessary qualification. This qualification amounts to saying that, having given one set of axes which will serve as a base for the statement of the theory, any other set which moves, relatively to the first, uniformly without rotation will do equally well. The qualification is however given, and, when we gather together all that is said on the subject, it is not easy to find any material flaw or omission. Having regard to the fact that Newton had in his mind the idea of the existence of an all pervading medium or ether, we are rather tempted to think that the notion of reference to such a medium tinged his views as to the base for space measurement. Professor Mach* says that Newton certainly had no such idea; but there does not seem to be any sufficient ground for this dogmatic assertion.

In a formal exposition of Newton's theory it is necessary to state, as one of the fundamental features of it, the conception of the division of matter into particles. Newton does not do this clearly; he contents himself with speaking of the parts of a body when he is dealing with one of which all points have not the same motion. His word "corpus" should often be translated "particle." This is a point which presents no difficulty.

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* *The Science of Mechanics* (American edition), p. 230.

Another matter as to which he is wanting in preciseness is his use of the word "force." Probably, however, it had not occurred to him to give an exact technical meaning to this word taken by itself. It is important to note that he commonly uses it in conjunction with some qualifying adjective or participle, such as innate, impressed, centripetal, accelerative, and motive. In fact some vague word, like influence or tendency, sometimes seems to be the best translation of Newton's word "vis".

The chief points laid down in the first chapter may be briefly summarized as follows:

(1) The mass of a body is measured by the product of its density and volume. It is no doubt intended to be understood that density is a property attached to each point of a body, independent of its surroundings so long as the body undergoes no change, and uniform in a body which other tests show to be homogeneous; accordingly this statement implies that the mass of a body is the sum of the masses of its parts. Thus the definition of mass, when completed incidentally in the following chapter, is to be subject to this proviso.

(2) The masses of bodies at a given place on the earth are found to be proportional to their weights.

(3) The quantity of motion (or the momentum) of a particle is measured by the product of its mass and its velocity.

(4) A force (or impressed force) acting upon a particle is somehow to be measured in terms of the motion of the particle relative to the base of reference adopted.

(5) The standard measurement of time is independent of the theory, and is based upon the comparison of the results of different repetition methods of measurement.

(6) The base, relative to which motion is to be reckoned in the statement of the theory, is to be defined by the test that all changes of velocity of particles relative to it are to be capable of expression in terms of impressed force.

The second chapter begins with the three laws of motion. Newton does not claim these as his own, and seems to have regarded them as having the advantage of being in a form which would meet with ready acceptance. The laws are worded as if the base of reference and the mass of a body had been completely defined. Thus the author escapes from the logical intricacies which have already been referred to. The position may be put as follows: These laws are approximately true for terrestrial motions relative to the earth, with masses measured by weights; and we now say that a base of reference can be so chosen, and masses so assigned to all particles of the universe, that they shall be accurately and universally true. This, together with what is contained in the previous definition of mass, is Newton's theory.

Let us go through Newton's three laws in detail. The rest and motion referred to in the first law are relative to the base which he has chosen, and has explained the characteristics of in terms of force. Impressed force has not been completely defined, so we must pass on at once to the second law. In the second law let us replace, in accordance with modern usage, change of motion (or momentum) by rate of change of momentum. This law then gives the relation between force, with its modern signification, and the corresponding rate of change of momentum of the particle upon which it acts relative to the chosen base. In the third law we come to the keystone of the theory, for it supplies the test by means of which forces are to be recognized, namely that they occur in pairs. Now we can work backward and see how the whole theory hangs together.

The intricacy of the theory is clearly seen when an attempt is made to compress it into one straightforward statement. It having been premised that the standard for time measurement is independent of the theory, such a statement might run somewhat as follows: Let all matter be conceived to consist of aggregates of particles, and to each particle let a constant numerical quantity be assigned to be called its mass. Let the vector quantity which is obtained from the velocity of a particle, relative to any axes, by multiplying the magnitude of the velocity by the mass of the particle be called the momentum of the particle relative to the same axes; so that the rate of change of momentum of a particle is similarly related to its acceleration. The rate of change of momentum of a particle can be decomposed into components by the parallelogram law in an infinite variety of ways. And the theory states that axes of reference can be so chosen, and the assignment of masses so arranged, that a certain decomposition of the rates of change of momenta, relative to the axes, of all particles of the universe is possible, namely one in which the components occur in pairs; the members of each pair belonging to two different particles, and being opposite in direction, in the line joining the particles, and equal in magnitude. Each component in the proposed decomposition is called a force, and is said to act upon the particle to which it belongs. Adopting the assignment of masses proposed, it is clear that their ratios alone concern us, that is to say, the mass of any one particle may be chosen arbitrarily. The sum of the masses of the particles composing a body is called the mass of the body ; and it is part of the theory that the masses of portions of a substance which is found by independent tests to be homogeneous are proportional to their volumes. It is clear that, if a set of axes relative to which the proposed decomposition is possible has been found, any other set which moves, relative to these, with uniform velocity and without rotation will do equally well. With the qualifications here stated, it may be regarded as part of the theory that the axes of reference, assignment of mass, and decomposition of rates of change of momenta into forces are unique.

Approaching the subject in this way it would be noticed that the application and verification of this complicated theory are rendered manageable by three important facts. The first is that we have at hand a series of approximations to axes with the properties required by the theory ; axes fixed to the earth being practically good enough for many cases of terrestrial motion, while axes with the center of mass of the solar system for origin and directions constant with regard to the fixed stars satisfy all ordinary requirements. The second is that laws of force of a permanent character can be ascertained by the methods of physics. We cannot indeed have experience of any single pair of forces, but can only obtain integral results each involving an infinite number of particles; these results, however, can be interpreted in terms of force according to the Newtonian theory. And the third fact is that at any given place on the earth the masses of bodies are found to be proportional to their weights.

If we assume that the theory is correct for the whole universe, and find that it holds for the relative motions of the parts of a given limited material system, the inference is that the forces between the particles of this system and those outside it correspond to accelerations which arethe same for all particles of the given system. And there is no reason why the relative motions of the bodies composing the given system should not be studied apart from other matter, a provisional base of reference being used. Accordingly, if we choose to enunciate the theory for the whole universe, any actual determination of an "absolute" base of reference should be regarded as provisional. In this connection some writers have said that absolute rotations are ascertainable, but not absolute accelerations.

Such a statement illustrates the objectionable character of the word absolute, and is likely to puzzle anyone who has not carefully studied the subject, but who knows that none but relative motions can be ascertainable.

The Scholium, which occupies the latter half of the chapter on the laws of motion, deals chiefly with the experimental verification of the equality of action and reaction, and shows how fully Newton recognized the fundamental character of this point. It is rather curious, considering how strongly the author has insisted on the distinction between his absolute motion and motion relative to the earth or any other body, that he refers to Galileo's use of the law of inertia, and discusses the experiments on collision made by himself and others without calling attention to the complications introduced by this point. It is indeed characteristic of Newton to credit his readers with sufficient intelligence to enable them to waive the mention of niceties which do not affect the argument in any practical way; but, at this particular stage, some reference to this matter would have been very opportune. The chief aim of the discussion of collision experiments is to show that they point to the equality of the momenta generated by the contact of the surfaces of solid bodies; and it is correctly, but tacitly, assumed that, to the degree of accuracy attained, the momenta measured may be regarded as "absolute" momenta. It is important to notice that the proof of the equality for attractions depends on the result having been already proved for pressures. The new experimental result, introduced for the case of attractions, is that a magnet and a piece of iron, floating in water, will rest in contact relative to the earth. The example of the attraction between two portions of the earth supplies evidence the value of which it is not so easy to estimate.

The remainder of the Scholium is only a statement of results. The author appears to see an analogy between the equality of momenta in collisions (*vis insita* being merely a measure of mass) and the equal and opposite amounts of work done, in a small motion, by balancing pressures applied to a machine; but no demonstrations are given. Results are enunciated in connection with work, which suggest a wider interpretation of the law of equality of action and reaction than that which necessarily belongs to the theory.

One of the most striking features of Newton's theory is the introduction of the base or axes of reference, relative to which it is claimed that all motion is capable of such simple statement. It may eventually become possible to establish a connection between this base and the ether; but for the present the axes introduced by the theory of kinetics belong only to that theory, and are not known to have any independent property. Accordingly such terms as "absolute motion" and "fixed axes," which may suggest irrelevant ideas, are rather objectionable for general use; and it would tend to clearness of statement if a name of a neutral character were adopted for the axes in question, or one which, if it connoted anything, should connote that they are mere creatures of the theory. The name "kinetic axes" is suggested as one which, in default of a better, would serve the purpose. The adoption of such a name would make it possible to avoid the awkwardness of speaking of two sets of axes as equally "fixed," in the technical sense, although each has a motion relative to the other. Moreover the term is one to which the word "provisional" can be conveniently prefixed when attention is to be called to the provisional character of the base which is employed.

Newton believed in the existence of an ether, or all pervading medium, the vehicle of the actions between portions of gross matter; but, knowing practically nothing about it, he excluded it from the system to which his theory was to be applied. In the concluding paragraph of the Principia he calls attention to the need for an investigation of its properties. At the present time the ether is included in any system to which the principle of Conservation of Energy is applied in physics, and Lagrange's equations are employed tentatively for the investigation of its behavior. But its position with reference to the Newtonian theory cannot be said to have been established, and the same remark applies to some extent to the case of molecular systems. In fact the theory appears to stand in need of at least some modification of form to enable it to meet the requirements of physics.

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