G_a is of the same form. Hence it remains only to consider the number of those which have no operator besides identity in common with G_a.

All these subgroups may be divided into two classes, viz: (1) those which are transformed into themselves by G_a, and (2) those which are transformed into different groups by the operators of G_a. The number of the former class may evidently be written in the form $ap + bq$, a and b being positive integers. If a group of the latter class occurs, all its operators must be commutative to every operator of G_a * and hence $r > p(q - 1)$. In this case the given theorem is evidently true. It may be observed that the number of self-conjugate subgroups of G is not necessarily of the given form, *e. g.*, the direct product of two non-commutative groups of order 21 contains only two self-conjugate subgroups of this order.

CORNELL UNIVERSITY,
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NOTE ON THE TETRAHEDROID.

BY DR. J. I. HUTCHINSON.

(Read before the American Mathematical Society at the Meeting of February 26, 1898.)

IN a brief paper, "A special form of a quartic surface," *Annals of Mathematics*, vol. 11, p. 158, I have called attention to an interesting special form of the locus of the vertex of a cone passing through six points. I wish to point out in this note the connection between this special surface and the tetrahedroid.

Given six arbitrary points in space 1, 2, 3, 4, 5, 6. These determine a system of ∞^3 quadric surfaces each of which pass through the six points. Denote this configuration by Σ.

Choose any arbitrary point P and consider the polar planes of P with respect to the system of quadrics. There are determined in this way ∞^3 planes forming a configuration Σ_1.

To a quadric in Σ corresponds a plane in Σ_1. The vertices of the cones of Σ have for locus a surface K of the fourth order. The planes of Σ_1 corresponding to the cones of Σ envelope a Kummer surface. The point in each plane corresponding to the cone vertex is the point of tangency.

* Dyck, *Mathematische Annalen*, vol. 22, p. 97.

To the twisted cubic k determined by the six basis points corresponds a single point O in $\sum_1$, which point is a node of the Kummer surface. To the fifteen lines 12, 13, ⋯, correspond 15 points (12), (13), ⋯. These are the remaining nodes of the Kummer surface.*

Suppose now that the six points 1, 2, ⋯, 6 form an involution on the cubic k, and that the lines 12, 34, 56, join the points that are paired in the involution. The quadric Q determined by these three lines will then contain the cubic k, and hence the corresponding plane in $\sum_1$ will pass through the four nodes O, (12), (34), (56) of the Kummer surface.

The Kummer surface accordingly becomes in this case a tetrahedroid.†

A quadric of $\sum$, since it contains six given points, may be required to pass through the three lines 13, 14, 23, being thereby completely determined. In the case of involution this quadric also contains the line 24. The corresponding plane in $\sum_1$ contains the four nodes (13), (14), (23), (24).

Similarly, the lines 35, 45, 36, 46, lie on a quadric of $\sum$, to which corresponds a plane containing the nodes (35), (45), (36), (46).

Finally, a quadric containing 15, 16, 25, 26, corresponds to a plane containing (15), (16), (25), (26).

An interesting question is suggested in this connection. It is well known that the Kummer surface is determined by six arbitrary points chosen for nodes. What relation exists among these nodes when the surface becomes a tetrahedroid? The answer is, *The six nodes form an involution on the twisted cubic determined by them.* I doubt whether this result can be proved in a simple manner from the correspondence established between $\sum$ and $\sum_1$. I will limit myself here to showing analytically the existence of this geometric property for the Fresnel wave surface (the projective equivalent of the tetrahedroid).

Taking the equation of the wave surface in the usual form and introducing a fourth variable w to make it homogeneous, consider, for example, the six nodes

$$(ib\beta,\ ia\alpha,\ 0,\ \pm\gamma),\ (c\gamma,\ 0,\ \pm a\alpha,\ i\beta),\ (\alpha,\ \pm\beta,\ \gamma,\ 0),$$

* A detailed study of the correspondences between Σ and Σ_1 is given by Reye; "Ueber Strahlensysteme zweiter Classe und die Kummer'sche Fläche," etc. (*Crelle*, vol. 86, pp. 84.)

† Cf. Cayley: "Sur un cas particulier de la surface du quatrieme ordre avec 16 points singuliers." (*Crelle*, vol. 65, p. 284.) Reye remarks (l. c, pg. 106) that the wave surface is apparently a special case (ein ziemlich specielles Beispiel) of the Σ, Σ_1 configurations, but carries his remark no further.

where

$$\alpha = \sqrt{b^2 - c^2}, \quad \beta = \sqrt{c^2 - a^2}, \quad \gamma = \sqrt{a^2 - b^2}.$$

Transform to a new system of coördinates x_1, x_2, x_3, x_4, by means of the equations

$$x_1 = (c - b)\,[a\alpha\beta x - (b\beta^2 - c\gamma^2)y + iac\alpha\gamma w],$$
$$x_2 = (b + c)\,[a\alpha\beta x - (b\beta^2 + c\gamma^2)y + iac\alpha\gamma w],$$
$$x_3 = (a + c)\,[a\alpha x - b\beta y + c\gamma z],$$
$$x_4 = (c - a)\,[a\alpha x - b\beta y - c\gamma z].$$

With respect to this new system, the coördinates of the six chosen nodes are (1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1), (e_1, e_2, e_3, e_4), (e_2, e_1, e_4, e_3), where

$$e_1 = \beta\,(b^2 - c^2)\,(c - a), \quad e_3 = (a^2 - c^2)\,(c + b),$$
$$e_2 = \beta\,(b^2 - c^2)\,(c + a), \quad e_4 = (a^2 - c^2)\,(c - b).$$

These six points form an involution of the kind described. (See *Annals of Mathematics*, vol. 11, p. 159.)

NOTE ON INTEGRATING FACTORS.

BY MR. PAUL SAUREL.

(Read before the American Mathematical Society at the Meeting of February 26, 1898.)

IF the differential equation

$$X_1dx_1 + X_2dx_2 + \cdots + X_ndx_n = 0, \quad n \gtreqless 3, \qquad \text{(A)}$$

be integrable, and if $u =$ constant be the integral of this equation, then, as is well known, there exists a function M such that

$$du \equiv MX_1dx_1 + MX_2dx_2 + \cdots + MX_ndx_n.$$

And as

$$\frac{\partial u}{\partial x_1} \equiv MX_1, \quad \frac{\partial u}{\partial x_2} \equiv MX_2, \cdots \frac{\partial u}{\partial x_n} \equiv MX_n,$$