If, however, the adjoined of any group G is discontinuous, G itself, and, of course, every group of the same structure is discontinuous. The bilinear form $\mathscr{F}_a$ is closely related to the adjoined group. In fact, if ϕ_a denotes the matrix of $\mathscr{F}_a$, the infinitesimal equations of the adjoined are

$$(a_1', a_2', \cdots, a_r) = (1 + \partial t \phi_a)(a_1, a_2, \cdots, a_r);$$

and we have
$$e^{\mathscr{F}\beta}e^{\mathscr{F}a} = e_{\mathscr{F}\gamma}$$

where
$$\gamma_j = \varphi_j(a_1, \cdots, a_r, \beta_1, \cdots, \beta_r) \qquad (j = 1, 2, \cdots, r).$$

CLARK UNIVERSITY,
    December, 1899.

PROOF OF THE EXISTENCE OF THE GALOIS FIELD OF ORDER p^r FOR EVERY INTEGER r AND PRIME NUMBER p.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, December 28, 1899.)

EXISTENCE proofs have been given by Serret[*] and by Jordan.[†] The developments used by Serret are lengthy but quite in the spirit of Kronecker's ideas. The short proof by Jordan, however, assumes with Galois the existence of imaginary roots of an irreducible congruence modulo p.

The proof sketched in this note proceeds by induction. Assuming the existence of the $GF[p^n]$, we derive that of the $GF[p^{nq}]$, q being an arbitrary prime number. Since the $GF[p]$ exists, being the field of integers taken modulo p, it will follow that the $GF[p^q]$ exists, and by a simple induction that the $GF[p^r]$ exists for r arbitrary.

We employ the lemma : *A factor of* $x^{p^{nm}} - x$, *belonging to and irreducible in the* $GF[p^n]$, *can be of degree* m' *if and only if* m' *divides* m. In particular, the irreducible factors of $x^{p^{nq}} - x$ are of degree q or 1. But the product of the distinct [‡] linear factors $x - \nu_i$ belonging to the $GF[p^n]$ is $x^{p^n} - x$.

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[*] Algèbre supérieure, 2, pp. 122–142.

[†] Traité des substitutions, pp. 16, 17.

[‡] Two functions belonging to the $GF[p^n]$ are called distinct if one is not the product of the other by a constant, a mark of the field.

Every such factor $x - \nu_i$ is a simple factor of $x^{p^{nq}} - x$. The statement being evident for the factor x, we assume $\nu_i \neq 0$, and proceed to prove that $x - \nu_i$ is not a factor in the field of

$$\frac{x^{p^{nq}-1} - 1}{x^{p^n-1} - 1} \equiv (x^{p^n-1})^{r-1} + \cdots + (x^{p^n-1}) + 1,$$

where $\qquad\qquad r \equiv (p^{nq} - 1)/(p^n - 1).$

But for $x = \nu_i \neq 0$, this function reduces in the $GF[p^n]$ to the value r, which is evidently not divisible by p.

It follows that the quotient $(x^{p^{nq}} - x)/(x^{p^n} - x)$ breaks up into factors of degree q belonging to and irreducible in the $GF[p^n]$. Any such factor may be used to define the $GF[p^{nq}]$.

MÉRAY'S INFINITESIMAL ANALYSIS.

Leçons nouvelles sur l'analyse infinitésimale et ses applications géométriques. Par M. Ch. MÉRAY, professeur à la Faculté des Sciences de Dijon. Paris, Gauthier-Villars. Première partie: *Principes généraux*, 8vo., 1894, xxxiii + 405 pp.; Deuxième partie: *Étude monographique des fonctions principales d'une seule variable*, 1895, xi + 495 pp.; Troisième partie: *Questions analytiques classiques*, 1897, vi + 206 pp.; Quatrième partie: *Applications géométriques classiques*, 1898, xi + 248 pp.

As a work of art—and mathematics is preëminently a fine art—this monumental work of Méray's is a masterpiece; as a treatise to induct students of mathematics into the mysteries of the infinitesimal analysis under the direction of any other than the author[*] its success is hardly so unqualified. Constructed so as to require technically as preliminary training nothing but a knowledge of elementary algebra and the theory of systems of linear algebraic equations, the first volume would indicate either or both of two conditions: that the author is a most marvelous teacher, that his pupils are of a race superior to that brilliant body of mathematicians now at Paris. It is one thing to be possessed of suffi-

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[*] The author states in the preface to the first volume (1894) that for twenty-four years he has found his method uniformly satisfactory in teaching.