necessary to frequent with assiduity the conferences, in order to acquire the dexterity necessary to solve the searching questions of these written examinations. Finally, there are the two theses already mentioned. As observed above, the principal thesis is ordinarily worked out quite independently by French students. For American students this is not necessary. They will find the professors under whom their work falls quite as ready to talk it over with them and to offer suggestions as in Germany.

Turn now to the doctorate of the university. Here the conditions are the same, except that two instead of three certificates are required. There is no question that the ordinary American student who takes his doctorate in two years in Germany could satisfy the requirements for the doctorate of the university of Paris in the same time; and as to their value as testimonials of work accomplished, I see no difference.

In closing this article I wish to express my thanks to all the gentlemen, too numerous to mention here, who by their courteous and kind attentions assisted me in procuring the materials for it. My gratitude is due in a particular measure to M. Paul Painlevé.

PARIS, *July*, 1899.

APPLIED MATHEMATICS.

Cinématique et Mécanismes, Potentiel et Mécanique des Fluides. Cours professé à la Sorbonne. Par H. POINCARÉ. Paris, Carré et Naud, 1899. 385 pp.

THE certificates granted at the Sorbonne for proficiency in mechanics require the study of a certain programme of subjects—amongst others, those on the title page of this volume. The somewhat curious mixture is probably accounted for by the fact that the students who attend these lectures desire to obtain only a certain degree of information and that in as short a time as possible. To treat four subjects in applied mathematics, within the compass of a single volume is of course impossible, if an adequate account of each is to be given. Nevertheless, M. Poincaré has, on the whole, succeeded in setting forth the main ideas of all of them, and his treatise thoroughly studied will probably fulfil the purpose which it is intended to serve.

The first chapter on kinematics contains the usual investigations of expressions for the motion of a point. As examples, the radii of curvature of a helix and of an ellipse are found directly from the dynamical formulæ. In the next chapter we are given the general kinematics of an invariable figure sliding on a plane. The lecturer goes to the root of the matter by considering the loci of the instantaneous center on the fixed and moving planes; the motion is obtained by rolling one of these curves on the other. The former locus is called the base and the latter the roulette, and M. Poincaré justifies the latter term by the property just mentioned. The name is somewhat unfortunate, as it is usually applied to the locus of a carried point and not to the rolling curve. Several examples are given, including the well known ones of the epi- and hypocycloids. The chapter is really one on geometry and the subject is treated as it should be treated, not as a part of the science of motion with the time element present, but as a purely geometrical science of motion.

The next stage is the motion of a body about a fixed point. As with the plane motion just treated, this can also be reduced to the rolling of one curve on another; but here the fixed and moving curves lie on a sphere with the fixed point as center. Hence, many of the propositions for plane curves can be applied to this extended case. In particular, spherical epicycloidal motion is of interest.

When the solid rotates about an axis and has a translation along the axis, we obtain helicoidal or screw motion. In the general case of the motion of a solid body, it is always possible and generally necessary to divide up the motion into a translation and a rotation. But the instantaneous motion can always be considered to be a uniform "screw" and the investigation thus becomes much simplified. The motion may also be considered as made up of two simultaneous rotations about two axes which do not meet, and the varying positions of these two straight lines as the solid moves constitute the main factor in determining its motion. A short chapter on relative motion and the theorem of Coriolis concludes this portion of the subject.

Under the heading "Mécanismes" M. Poincaré treats the geometrical aspects of machines for transmitting motion. The principal problems to be solved are the transformations of continuous or alternating circular motion into continuous or alternating circular or rectilinear motion. The machines for this purpose can be generally divided into two classes—transmission by hinged bars and that by continuous contact. The geometrical problem to be solved in the latter case is

this :—Knowing the motion of both parts and the form of one part, to find that of the other. Since the parts are in contact, the problem reduces to the discovery of the contour of the second part. This is obtained by finding the envelope of the successive positions of one part as seen by an observer rigidly attached to the other. It is thus merely a problem in geometry or rather in kinematics. In most cases where toothed wheels are not present the problem is quite simple and is easily solved as soon as the two motions are given. But when wheels with teeth are used we have to determine the form to be given to the teeth, and further, the form most effective for the particular machine to which they belong. M. Poincaré goes into those points with some detail. He discusses the simplest case of two wheels of different diameters and in the same plane, by means of the work in the earlier chapters, especially that on epicycloidal motion. Then follow more complicated cases, including those where the wheels are not in the same plane. Finally several pages are devoted to eccentrics and other forms of transmission.

The chapters on potential are excellent. The earlier part is perhaps a little condensed, but it contains all that is necessary. Then follow Green's theorem and many applications ; and the final chapter is one on the attraction of an ellipsoid, where full use is made of the theorems tending to shorten what may be a long and troublesome piece of analysis.

In hydrostatics, after the usual preliminaries, most of the investigations are concerned with the action of liquids under gravity only. A complete solution of the barometric formula connecting the difference of height of two places with the difference of the barometric readings is given, with numerical results. M. Poincaré then deals with the metacenter of a floating body, which again gives him an opportunity to proceed mainly by geometrical methods, and of this he takes full advantage.

The chapter on hydrodynamics is rather out of proportion in its various parts. One would hardly think that the purpose of the teacher is achieved when wave motion is confined to one short article and the theory of vortices is gone into with some detail. In fact, there are 14 pages in which the general equations are briefly discussed, 7 pages on the efflux of liquids from a containing vessel, 4 pages on small movements of liquids (waves), 25 pages on vortices. Then follow 9 pages on the motion of a solid in a liquid, most of which are taken up with the problem of the mutual effect of two pulsating spheres. But if doubt is expressed as to

the reasonableness of M. Poincaré's exclusion or insertion of matter, none can be entertained of the excellence of his manner of exposition. Without long circumlocutions and yet with comparatively few symbols, he gets right into the heart of the problems he is discussing : it may be that he has chosen them from this point of view. No better example of his method can be furnished than the way he reaches the action of two rectilinear vortices on one another. The reading of this will appeal strongly to the student and whet his appetite for more information. In all the hydrodynamical part—and, indeed, in the rest of the book—practically the only results of an at all advanced nature that a student needs are those deduced from Green's theorem. But these have already been dealt with in the chapters on potential. An instructor who wishes to give a short course on hydrodynamics can hardly have a better model for the details than this chapter, and he will not find it difficult to add to it the portions which are necessary in order that his hearers may obtain a general idea of the problems which arise in the subject.

ERNEST W. BROWN.

SHORTER NOTICES.

Annuaire pour l'An 1900, publié par le Bureau des Longitudes. Paris, Gauthier-Villars.

THIS handy little volume, brought out for popular and professional use, is as usual improved by the addition of new matter, the omission of portions of no special value, and the alteration of details here and there. Among the additions may be noticed tables of the right ascension of the sun at mean noon and of the right ascension, declination, and parallax of the moon, together with some auxiliary astronomical tables. The magnetic elements for the principal towns in France have been brought down to January 1, 1900. In the "Notices," M. Janssen gives his annual report of the work done at the observatory on the summit of Mont Blanc ; he also contributes a note on the use of balloons for astronomical purposes. M. Lippmann describes briefly but clearly the discovery and main properties of the newly found atmospheric gases. The longest article is on the theory and construction of dynamos, and this deserves special mention. As is usual with French writers, M. Cornu be-