any sincere writer as totally illogical in itself, and baneful in its effects upon the student.

While this is but one phase of the book in hand, it is the most important phase, to which all else is made subordinate. We will then note but one other point : namely, that the author protests against a violation of the axiom that two points always determine a straight line (page 44), but assumes that " a whole is greater than a part " even when the " whole " in question is infinite (page 19). In both particulars the modern tendency is certainly quite the opposite.

The philosophical portion of the book will not be criticised here, but it is scarcely felt that a philosopher would care to be sponsor for the statements made — certainly not for the style in which they are presented. The philosophical views of mathematical thought, and in particular of infinity and infinitesimals, must surely take into account, however, the positive results now in the possession of mathematicians regarding the effect of the violation of the archimedean axiom upon our system of axioms and upon our conceptions of space.

<div align="right">E. R. HEDRICK.</div>

SHEFFIELD SCIENTIFIC SCHOOL,
        November, 1902.

SOME RECENT GERMAN TEXT-BOOKS IN GEOMETRY.

Lehrbuch der Elementar-Geometrie. Erster Theil, Planimetrie. Von Dr. E. GLINZER. Siebente Auflage. Dresden, Kühtmann, 1899. 4to, 120 pp.

Grundriss der Geometrie. I. *Planimetrie.* Von J. H. KÜHL. Zweite Auflage. Dresden, Kühtmann, 1900. 4to, 95 pp.

Lehrbuch der Stereometrie. Von Dr. P. SAUERBECK. Stuttgart, Kröner, 1900. 4to, 291 pp.

Die Elemente der analytischen Geometrie. Zweiter Theil: Die analytische Geometrie des Raumes. Von Dr. F. RUDIO. Dritte, verbesserte Auflage. Leipzig, Teubner, 1901. 4to, vi + 186 pp.

THE authors of the first two texts are connected with the institution in Hamburg known as the Allegemeine Gewerbeschule und Baugewerkschule. One might expect accordingly

a general disregard of anything approaching euclidean rigor in their treatment of the subject. In fact, no precise statement of axioms or definitions is anywhere to be found in either book. The parallel axiom, as such, does not appear at all.   Dr. Glinzer gives Euclid's form, viz.: If two lines are cut by a third so that the sum of the interior angles on one side is less than a straight angle, these lines will, if sufficiently produced, meet on that side — as a *proposition*, the proof following it involving comparison of infinite areas.   The trivial nature of this proof is, however, emphasized by the setting in small type.   The other author assures the pupil that the proof of the proposition : If two lines are cut by a third under equal alternate interior angles, the lines are parallel, does not necessarily imply the truth of the converse, but the correctness of the latter is easily established graphically.

Irrational numbers receive scant attention, the propositions involving incommensurable lengths being regarded as relatively unimportant.   The word "limit" is not mentioned at all, but the pupil is told off-hand to apply the formulæ for regular polygons to the case when the number of sides become infinite.

All of this goes to show that, in this German industrial school, the facts of geometry are to be learned at any cost, the development of the student's logical faculties being a matter of secondary importance.   A geometric sense must be cultivated and the feeling instilled in the learner that drawing a figure is often half the battle in proving a theorem in geometry.   To this end, much attention is paid to constructive geometry, a feature worthy of imitation in American texts.

In extent the subject matter is much the same as the traditional plane geometry.   Type and figures are clear, but hardly up to the standard of our best American books.

In the preface to Dr. Sauerbeck's volume the author registers his opinion that "the first object of solid geometry is the cultivation of space intuition."   With this view the reviewer is heartily in agreement, and he believes that the usual American text-book contributes very little to this end.   For the stress is felt by the student to be laid on the *form* of proof, and not on the *content* of the proposition.   After a strenuous course in plane geometry, it may be assumed that euclidean form and rigor in demonstration have been acquired, and in the remainder of the course, emphasis should be put not on logical develop-

ment and sequence of propositions, but on the facts of solid
geometry and spatial relations. The traditional course in
American colleges supplementing the usual requirement in
plane geometry for this reason results in a paltry addition to
the sum of the student's knowledge of geometry. How much
greater would be the gain in power if an amount of ground
could be covered similar to that presented in Dr. Sauerbeck's
text! For in pages 1–59 the author discusses the usual
theorems involving metrical relations of planes and lines, but
in addition such of the elements of projective geometry as
naturally arise in the consideration of central projection, the
principle of duality, etc. Next follows a section on crystal-
lography, well done but of doubtful value. Section V, pages
84–133, treating of the polyhedral angle and the sphere, is quite
complete, including radical planes, axes, centers of similitude,
etc. Following this (pages 133–211) surfaces of revolution are
discussed, and in this section the subject of conic sections finds
consideration, together with stereographic projection and appli-
cations. The closing pages treat of mensuration. In these the
theorem of Cavalieri is assigned its proper place beside the
method of exhaustion. One wonders why this extremely simple,
evident and powerful theorem has won no place in American
texts. The theorems of Guldin are proven, but seem out of
place, at least in their general form. The very general class of
solids known as Simpson's solids furnish an excellent example
in cubature, and give, as analogous formulæ in the plane, the
well-known Simpson's rule for quadrature. A commendable
feature of the book is the large number of exercises, solved and
unsolved.

To repeat, the position of the author seems to the reviewer
a very strong one, and it is to be hoped that the coming years
will show a departure from the present prevalent practice of
teaching solid geometry in our colleges after the manner of the
secondary schools.

Dr. Rudio's book, with its companion volume on the plane,
is deservedly popular in Germany. Evidence of great care and
thoroughness in writing is everywhere abundant throughout
the volume, and the result is a most excellent text. The prin-
ciple of projection is adopted most wisely as fundamental, and
the fixing of the algebraic sign of lengths, areas and volumes
carefully treated. The author has kept strictly within the

limits set by the title, the book closing with a discussion of the elementary properties of quadrics, such as tangent planes, diameters, rectilinear generators, etc. The collection of exercises is large, and figures and type are clear and attractive.

PERCEY F. SMITH.

SHEFFIELD SCIENTIFIC SCHOOL,
December, 1902.

NOTES.

THE opening (January) number of volume 4 of the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY contains the following papers: "Orthocentric properties of the plane n-line," by FRANK MORLEY; "Definitions of a field by independent postulates," by L. E. DICKSON; "Definitions of a linear associative algebra by independent postulates," by L. E. DICKSON; "Two definitions of a commutative group by sets of independent postulates," by E. V. HUNTINGTON; "Definitions of a field (Körper) by sets of independent postulates," by E. V. HUNTINGTON; "On the invariants of differential forms of degree higher than two," by C. N. HASKINS; Ueber die Reducibilität der Gruppen linearer homogener Substitutionen," by ALFRED LOEWY; "The quartic curve as related to conics," by A. B. COBLE; "The cogredient and digredient theories of multiple binary forms," by EDWARD KASNER; "On the envelopes of the axes of a system of conics passing through three points," by R. E. ALLARDICE; "A Jordan curve of positive area," by W. F. OSGOOD.

THE January number (volume 25, number 1) of the *American Journal of Mathematics* contains: "The parametric representation of the tetrahedroid surface," by D. N. LEHMER; "On ternary monomial substitution groups of finite order with determinant ± 1," by E. B. SKINNER; "On forms of unicursal sextic scrolls," and "On forms of sextic scrolls of genus one," by VIRGIL SNYDER; "Note on symmetric functions," by E. D. ROE, Jr.

THE number contains a portrait of Professor L. CREMONA.

THE lectures delivered by Professor OSKAR BOLZA before the colloquium of the AMERICAN MATHEMATICAL SOCIETY at Ithaca, N. Y., in August, 1901, will be published early in the