THE FEBRUARY MEETING OF THE SAN FRANCISCO SECTION.

THE seventh regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held at Stanford University, on Saturday, February 25, 1905. The following members were present:

Professor H. F. Blichfeldt, Professor G. C. Edwards, Mr. G. I. Gavett, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor A. O. Leuschner, Professor G. A. Miller, Professor H. C. Moreno, Dr. B. L. Newkirk, Professor C. A. Noble, Professor T. J. J. See, Professor Irving Stringham, Professor A. W. Whitney.

The attendance also included a number of teachers of mathematics and physics who are not members of the Society. A morning and an afternoon session were held, Professor Haskell acting as chairman at both sessions. It was agreed to hold the next meeting at the University of California on September 30, 1905.

The following papers were read at this meeting:

(1) Professor H. F. BLICHFELDT: "On a theorem due to C. Jordan."

(2) Professor H. F. BLICHFELDT: "On the order of the collineation groups in five variables."

(3) Professor A. W. WHITNEY: "A theorem in the theory of probabilities, and its application to insurance."

(4) Professor R. E. MORITZ: "A general theorem on local probability."

(5) Professor E. J. WILCZYNSKI: "Projective differential geometry of plane curves."

(6) Dr. W. A. MANNING: "On the primitive groups of class ten."

(7) Professor G. A. MILLER: "Invariant subgroups of prime index."

(8) Professor IRVING STRINGHAM: "A geometric construction for quaternion products."

(9) Professor A. O. LEUSCHNER: "On the general applicability of the short method of determining orbits from three observations."

(10) Professor T. J. J. SEE: "On the physical state of the matter of the earth's interior, with considerations on terrestrial geology, and on the comparative geology of the other planets."

In the absence of the authors, the papers of Professors Moritz and Wilczynski were read by Professors Leuschner and Haskell respectively; Dr. Manning's paper was presented by Professor H. C. Moreno. Professor Stringham's paper will appear in a later number of the BULLETIN. Abstracts of the other papers are given below. The abstracts are numbered to correspond to the titles in the list above.

1. The theorem due to C. Jordan, that the order of a linear homogeneous group G in n variables is of the form λf, where f is the order of an abelian self-conjugate subgroup of G, and where γ is inferior to a fixed number depending only upon n, has been proved for a class of linear homogeneous groups called primitive by Professor Blichfeldt (*Transactions*, volume 4, page 387; volume 5, page 310), a number having been found that λ must divide. In the present paper, Professor Blichfeldt proved the corresponding theorem for imprimitive groups of the kind considered, thus completing his proof of Jordan's theorem for all transitive ("irreducible") groups, from which the theorem is readily demonstrated for groups in general.

2. Professor Blichfeldt's second paper is devoted to a proof of the following theorem : In a primitive group G in 5 variables, containing no invariant subgroup and not being identical with the group H (cf. *Transactions*, volume 5, page 315), the number λ must be a factor of one of the following numbers : $2^7 \cdot 3^5 \cdot 5^5$, $2^7 \cdot 3^3 \cdot 5^3 \cdot k$, where k is one of the numbers 7^2, $7 \cdot 11$, 11^2, $11 \cdot 13$, 13^2, 17 or 19.

3. Professor Whitney proposed the ratio of the expected loss to the probable fluctuation as an index of the security of the profits from being used in meeting exceptional losses. From this index he calculated the actual probability that they will be thus exhausted in any year. The condition for this index to be a maximum yields a criterion for the best distribution of lines. The method readily lends itself to the consideration of partial loss. It is furthermore entirely susceptible of purely statistical treatment.

4. Czuber, in his Geometrische Wahrscheinlichkeiten und Mittelwerte, shows that if on a straight line of length a two points are assumed at random, the probability that their dis-

tance apart exceeds a given length b is $P = [(a - b)/a]^2$. Professor Moritz extends this theorem to the case of the probability that of n points distributed at random along a line of length a, no two shall fall within a distance b of each other. If $x_1, x_2, \cdots, x_n$ represent the respective distances of n points from one extremity of the line a, and if Q represents the probability that no two points fall within a distance b of each other when the points are subject to the conditions $x_1 < x_2 < \cdots < x_n$, then $P = n! \; Q$, where

$$Q = \frac{1}{n!}\left(\frac{a - \overline{n - 1}b}{a^2}\right)^n, \quad P = \left(\frac{a - \overline{n - 1}b}{a}\right)^n.$$

For the case of three points the theorem can be proven geometrically. One set of concurrent axes of a cube with edge a are taken as axes of coördinates. A one to one correspondence is seen to exist between the coördinates x, y, z of any point within the cube and the respective distances x_1, x_2, x_3 of three points on a line of length a. The problem now consists in comparing the volume of the entire cube with the volume of the regions for which simultaneously

$$x \sim y > b, \quad y \sim z > b, \quad z \sim x > b.$$

This gives for the required probability $P = [(a - 2b)/a]^3$.

From the general proportion follows the important corollary : The probability that of n points, distributed at random on a closed curve of length a, no two shall fall within an arc b of each other is $[(a - nb)/a]^{n-1}$.

5. Professor Wilczynski presents in a simpler and clearer form the results of Halphen's thesis "Sur les invariants différentiels." The paper forms at the same time the conclusion of the author's investigations on differential projective geometry, so far as it concerns curves and ruled surfaces. All of the papers which touch upon this subject will be shortly combined into a systematic treatise on the projective differential geometry of curves and ruled surfaces. It is the author's intention after the completion of this treatise to take up the general theory of surfaces from the same point of view, thus creating for projective geometry a general theory which shall correspond to the place filled by the Gauss-Monge theory in metrical geometry.

Although primarily a new rendering of Halphen's results, this paper is in form entirely different from Halphen's thesis,

and contains many results and points of view not to be found in Halphen's memoir. Still, so far as it has been possible to do so with the different treatment adopted, the author has confined himself to the scope of Halphen's thesis, so as to make more convenient the comparison of the two methods.

6. Four of the primitive groups of class 10 contain substitutions of order 5 on 10 letters. These are the metacyclic group of degree 11, and the Mathieu group of degree 12, and their positive subgroups. This result was given by Dr. Manning in volume 4 of the *Transactions*. In the present paper he completes the list of the primitive groups of class 10 by adding three groups which do not contain this substitution of order 5. One is the diedral rotation group of order 22, one the $G_{7!}^{21}$, simply isomorphic to the symmetric group of degree 7, and one is a new group of degree 25 and order $2(5\ !)^2$. This last forms part of a system of primitive groups of degree k^2, order $2(k\ !)^2$, and class $2k$ $(k > 2)$.

7. The number of invariant subgroups of index p in any group G is of the form $(p^\lambda - 1)/(p - 1)$. When G is abelian λ is equal to the number of invariants in the Sylow subgroup of order p^m contained in G. The determination of all the possible groups of order p^m in which λ has a given value is quite difficult except in a few cases. When $\lambda = 1$ such a group is cyclic and when $\lambda = m$, G is the abelian group of type $(1, 1, 1, \cdots)$. All the possible groups in which $\lambda = m - 1$ are also known. Professor Miller's paper is devoted to the case where $\lambda = m - 2$. The investigations are simplified by the use of the theorem that only invariant operators can be common to every subgroup of index p of a group of order p^m whose commutator subgroup is of order p. The paper will be offered to the *Transactions* for publication.

9. After a brief presentation of the fundamental principles of his "Short method of determining orbits from three observations" without hypothesis regarding the eccentricity, Professor Leuschner developed the formulæ necessary to determine whether, in a given case, the solution is determinate, indeterminate, or partially determinate. He discussed the special cases to which one or the other of the older methods are inapplicable. In particular, it was pointed out that with the "short method" the orbit does not necessarily become indeter-

minate when the three geocentric places and the second solar place lie in a great circle, the following three additional conditions being necessary to make the solution indeterminate in this case : (1) The intervals must be equal; (2) the third powers of the intervals multiplied by the solar constant k must be numerically ineffective ; and (3) the heliocentric distance must be comparatively large.

The " short method " is particularly suitable for correcting preliminary elements by means of a longer arc. For this purpose, the geocentric distance and the velocities in the heliocentric rectangular coördinates are first derived for the middle date from the ephemeris or from the constants of the preliminary orbit and then corrected on the basis of the residuals of the first and third observations. Modifications of the "short method " were given to enable a computer to derive (1) the range within which the period of the orbit may lie, and (2) a parabola directly in case the parabola should be included within this range.

10. The purport of this paper was to show that the matter within the earth and other large planets is genuine gas above the critical temperature of every known substance, yet so compressed as to behave with more perfect elasticity than any solid. This view had been reached by the study of the pressures within the other planets, which Professor See has treated in *Astronomische Nachrichten* no. 3992. He submitted calculations to prove that under the immense pressures existing in the heavenly bodies porosity and interpenetrability are general properties of matter, and that all matter thus becomes fluid, the molecular forces entirely disappearing in comparison with the strains to which the matter is subjected. He also showed that if the planets were made of the stiffest granite or steel they would assume the globular form from pressure alone. The spheroidal figures of the planets not only prove their original fluidity, but the fluidity of all matter under planetary pressure, which is reckoned in millions of atmospheres.

As regards the earth he showed that at a depth of 63.7 kms. matter released from the imposed pressure would be crushed, fused and vaporized in the process, and in this way explained volcanic phenomena. Though the earth's matter behaves as a solid, on account of its imprisonment, it is really elastic vapor reduced to great hardness by pressure, and if the pressure were

relieved, as by sinking a vacant shaft down to it, a violent explosion would follow.

The author concluded from Laplace's law of density that the planet Venus has a rigidity equal to that of a corresponding globe of glass, and that the nuclei of all large bodies are effectively of the highest rigidity. The development of pressure as we decend in the globe was shown to be such as to invalidate the conclusions of Lord Kelvin and Professor G. H. Darwin respecting the consolidation of earth by the building up of a solid nucleus from the sinking of the solidified crust.

G. A. MILLER,
Secretary of the Section.

ON THE DEVELOPMENT OF MATHEMATICAL ANALYSIS AND ITS RELATION TO CERTAIN OTHER SCIENCES. *

BY PROFESSOR EMILE PICARD.

ONE of the objects of such a congress as that in which we are now assembled is to show the connection between the different parts of science taken in its widest sense. Moreover, the promoters of this meeting have insisted that the relations between different branches should be put in evidence. To undertake a study of this kind, the character of which is somewhat indefinite, one must forget that all is in all ; as for algebra and analysis alone, a Pythagorean would be dismayed at the extent of his task, remembering the celebrated formula of the school : "Things are numbers." From this point of view, my subject would be inexhaustible. But for excellent reasons I should not attempt so much. Merely glancing at the development of our science through the ages, and particularly in the last century, I hope to be able to characterize sufficiently the

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