scarcely find a proper place for review in this periodical. Besides, it should appear that even if one man did write of all these matters, one reviewer could not be expected to treat all of them in other than the perfunctory way in which almost all offerings of scientific work are "reviewed" in the daily or weekly press.

E. B. WILSON.

Algebraic Equations. By G. B. MATHEWS, M.A., F.R.S., Fellow of St. John's College. Cambridge University Press, 1907. 64 pp.

THIS is one of a half dozen "Cambridge Tracts in Mathematics and Mathematical Physics," issued by the Cambridge University Press. In this tract the author is confronted with the problem of giving an exposition of the Galois theory of equations within the compass of about sixty pages. This limitation makes it necessary for him to present the subject more or less in outline and to confine himself to a very few illustrative examples. An outline presentation, prepared by an eminent author, is certain to bring out in bold relief interesting view points. Such is the case in this booklet. And yet we are of the opinion that the real value of this book to beginners would have been enhanced by more abundant illustration and a somewhat fuller detail of explanation.

To save space, the author does not put down a definition in a sentence by itself; the definition is to be inferred from a condensed statement made as part of a sentence occurring perhaps in the body of a demonstration. Thus, the definition of an intransitive group (page 14) is given in course of a proof, as follows: "First suppose G is intransitive: this means that a certain number of roots $x_1, x_2, \cdots, x_r$ $(r < n)$ are only interchanged among themselves by the substitutions of G." Less easily comprehended are the definitions of simple groups and self-conjugate factor groups, similarly interpolated on pages 16 and 17. Despite the effort to secure extreme condensation, there occur redundancies, such as "absolutely undetermined" (page 2), "absolutely unaltered" (page 57), "perfectly definite" (page 6). These are less objectionable in oral exposition than in a printed outline.

Here and there are evidences of hasty composition. Thus, the tract is encumbered with some heterogeneous terminology. The author speaks in different places of the "arithmetically rational," and the "rational in the ordinary sense," also of a "definite numerical irrationality" and an "irrational quantity." The terms "rational" and "rational function" are not always accompanied with an explicit statement of the field of rationality, so that extra care must be exercised by the reader, lest he misinterpret the author. On page 5 is proved the theorem that any rational function of the roots of a given equation $f(x) = 0$ can be expressed as a rational function of any one of the roots of its complete galoisian resolvent. This is done at a time when the meaning of the term "rational" has not, as yet, been extended to apply to different fields. However, the generalization is made on the next page. As a corollary to this theorem, it is shown that all the roots of a complete galoisian resolvent may be expressed as rational functions of any one of them; an equation having this property is called a normal equation. As this complete galoisian resolvent may be reducible in the field defined by the coefficients of the given equation, it would follow that a normal equation may be reducible — a definition that is both undesirable and contrary to the usage of writers on the Galois theory.

An unfortunate phraseology occurs on pages 2, 3 and 4, in connection with "arbitrary" and "numerical" coefficients. It leaves the impression that equations of higher degrees than the fourth are algebraically unsolvable only when the coefficients are "arbitrary." "If $n < 5$ and the coefficients are left arbitrary, it is possible to construct an explicit algebraic function of the coefficients which is a root of the equation. For $n > 4$, this is no longer the case." As if this statement did not apply at all to equations with numerical coefficients, the author proceeds to say: "When the coefficients are numerically given," the roots can be found by trial or approximation, although the main problem with him is this: "Given a particular equation with numerical coefficients, it is required to find the simplest set of irrational quantities such that all the roots of the given equation can be expressed as finite rational functions, in an explicit form, of the set of irrationals." Returning to arbitrary coefficients, he gives then a preliminary discussion of the cubic, quartic and quintic. Nowhere on these pages does he convey the idea that numerical equations may be algebraically unsolvable, nor does he here make plain that, besides the extreme cases of coefficients that are all "arbitrary" or all "numerical," there are intermediate assumptions which demand our attention. Needless to say, no one knows the facts in question better than our distinguished author, and it is a pity that he did not take time to express himself more precisely. Later the author drops the term "arbitrary coefficients" and speaks of "coefficients represented symbolically" (page 11).

The foundation of the Galois theory covers in this tract the first twenty-nine pages; then eleven pages are given to cyclic equations, four pages to abelian equations and ten pages to metacyclic equations. The last chapter is upon "Solution by standard forms" and contains, on six pages, in outline, the treatment of the quintic by means of the icosahedral irrationality. This outline can be used with interest in conjunction with Klein's Ikosaeder.

FLORIAN CAJORI.

Introduzione alla Geometria Proiettiva degli Iperspazi. By E. BERTINI. Pisa, Spoerri, 1907. 425 pp.

THE subject of geometry of n dimensions has undergone a noteworthy advancement during the last few decades. In this advancement the Italian geometers have taken a very prominent part. It is therefore fitting that this able work on projective geometry of n dimensions should be the work of an Italian.

The author avoids the difficulties of spacial intuition by starting from a purely arithmetical definition of a point and of a space. There exist, in ordinary geometry, numerous examples of systems of entities which can be put into one to one algebraic correspondence, without exceptional elements, with the values of the ratios of a system of parameters. The properties of such systems are of two kinds, those depending on the particular nature of the entities and those depending only upon the fact of the correspondence with the group of values of the parameters. The latter properties the author considers to form the subject matter of projective geometry of n dimensions. A point is therefore defined as a set of ratios of $n + 1$ parameters $x_0, x_1, \cdots, x_n$, and a space of n dimensions as "the totality formed by all the sets of values (real and complex) of these ratios."

The first two chapters are devoted to a discussion on this basis of the ideas of subordinate spaces, section, projection, etc. The possibility of a satisfactory synthetic treatment is also pointed out, however, and a number of references to such treatments are given.

The next three chapters, of about eighty-four pages, are de-