Then with this condition satisfied variations of **E** which vanish at S_0 but not at the surfaces S_i affect only the second term of U and give the condition $V = V_i$, making the potential constant over the surface of each single conductor ; and finally variations which at the surfaces S_0 are restricted only by the boundary condition (II) give the condition $V' = V''$, making the potential continuous even at surfaces where its derivatives may be discontinuous.

CHICAGO, ILL.,
May, 1908.

THE FOURTH INTERNATIONAL CONGRESS OF MATHEMATICIANS.

THE fourth international congress of mathematicians was held at Rome, April 6 to 11, 1908, under the efficient management of the Circolo Matematico di Palermo and under the patronage of His Majesty, the King of Italy. Including the ladies who accompanied the members of the congress, the enrollment was more than seven hundred. The list contains the names of the following Americans: Miss E. M. Coddington, H. W. Curjel, E. W. Davis, T. S. Fiske, A. B. Frizell, W. J. Graham, J. G. Hardy, E. A. Harrington, A. S. Hawkesworth, T. F. Holgate, A. Macfarlane, Artemas Martin, C. L. E. Moore, E. H. Moore, Simon Newcomb, G. D. Olds, G. B. Pegram, D. E. Smith, J. M. Van Vleck, W. D. A. Westfall.

The general order of the program provided for sectional meetings in the morning and general conferences in the afternoon. However this order was broken occasionally. The arrangements of the committee on entertainment left nothing to be desired.

The first meeting of the members of the congress was at the reception offered by Professor Tonelli, Rector of the University of Rome, Sunday evening, April 5. This was the beginning of the social part of the occasion. Those who attended the congress will always have pleasant recollections of these receptions and other entertainments. The opening reception was held in the library of the University. The Mayor of the city of Rome, Mr. Nathan, was present. Acquaintances which were to broaden during the week began here. Excellent refreshments were served, and the mathematicians showed that they were not incapable of enjoying this part of the program.

The congress officially opened on Monday, April 6, at a meeting held at the Capitol, in the presence of His Majesty, King Vittorio Emanuele III. The meeting was opened by Mayor Nathan, who welcomed the congress in the name of the city of Rome. The minister of public instruction, Rava, extended a welcome on behalf of the government of Italy. President Blaserna of the committee also made an appropriate address. The session closed with the address of Professor V. Volterra on: "Le matematiche in Italia nella seconda metà del secolo XIX."

The lecturer began by recalling the period of Italian political rejuvenation, in which the whole life of the nation was renewed and the universities reopened. It was then that the instruction in higher mathematics was instituted and entrusted to such mathematicians as Brioschi, Betti, Cremona, Fergola, and Battaglini. Very soon after these followed other mathematicians of no less eminence. The period of original and scientific research in Italy may be said to date from that time. From that moment different Italian schools of mathematics began to be created and to develope.

After drawing a brief comparison between the Italian studies in the first and the second half of the nineteenth century, in order to throw light on the substantial diversity of the surroundings in which they developed, the lecturer passed in review the different Italian mathematical schools, showing their tendencies and their scope.

On account of their research in mathematical physics, which principally absorbed the attention of Betti and Beltrami, he called them the champions of mathematical physics in Italy. He examined the advances in electricity initiated by Betti and continued by a series of other mathematicians, and the researches of Beltrami, who can be placed among those who investigated in a systematic manner the perturbations introduced into the equations of mathematical physics by the hypothesis of a curvature of space.

After having mentioned other investigators in mathematical physics and mechanics, the lecturer passed to another order of investigation which was developed in Italy, viz., that of the theory of functions and allied subjects in analysis. He recalled, in this connection, his address at the Paris Congress on Betti, Brioschi, Casorati, and on the different methods in which these three mathematicians (who were the initiators in Italy of the study of the theory of functions) conceived the theory itself. He recalled also the various investigators of analysis and dwelt more especially on a branch of research which flourished in Italy a little apart, and which was somewhat forgotten for some years, but which recently acquired general interest and attention, that is research in functions of real variables and their singularities. Dini introduced and diffused their study in Italy and put this theory at the basis of his instruction in the infinitesimal calculus. It then took a double direction, one conducted to research in the field of pure mathematics while the other led to studies which acquired a more philosophic character.

From these researches the lecturer passed to those which, in Italy, are wont to be called geometric. He spoke of the singular struggle, which manifested itself more acutely in Italy than elsewhere, between those who were called analysts and those who were called geometers, but demonstrated the unreality of the distinction which was usually made, and the equivocation on which it was founded. In these days the distinction can only be called a relic of the past.

He examined next the work of Cremona and of his students and disciples, and observed that, forty years after Cremona had begun his teaching, Klein affirmed that Italy had become the real center of geometric research, although this field had been almost ignored in Italy until Cremona initiated his course in higher geometry. The lecturer also examined the researches in hypergeometry, algebraic geometry, and especially the geometric interpretation of the theory of forms. He showed how the most recent Italian geometric investigations, being connected with those of Picard on the theory of algebraic functions, reenter the sphere of the theory of functions. He spoke also of infinitesimal geometry, in which much research has been carried on in Italy, and which makes a noble counterpart to the work in pure and algebraic geometry.

Finally, the history of mathematics developed in the last fifty years was recalled, and the critical publications on the works of Galileo were mentioned.

The lecturer closed by showing that the present time in Italy presents some analogies with a half century ago, and that now, as then, a reform in the program of mathematical studies is necessary. These, in fact, in Italy, are identified with the schools of engineering, for which he urged a more modern organization. He concluded by expressing the hope for a continuous development in Italian mathematical thought harmoniously united with that of other nations.

In the afternoon the congress met at the Palazzo Corsini and organized. Professor Blaserna was chosen as permanent president, after which the various other officers were named. The first business of the session was the report, presented by Professor Segre, of the committee on the Guccia Medal. There were three competing memoirs, but for various reasons these were adjudged unworthy of the prize. The committee therefore examined the literature published during the time from November 1, 1904, to July 1, 1907, and decided that the prize should go to Professor Francesco Severi for his investigations on the geometry on an algebraic surface. The publications of Professor Severi referred to are the following:

1. "Sulle superficie algebriche che posseggano integrali di Picard della 2^a specie" (*Mathematische Annalen*, volume 61).

2. "Sulla differenza tra i numeri degli integrali di Picard della 1^a e 2^a specie appartenenti ad una superficie algebrica" (*Atti di Torino*, volume 40).

3. "Sulla totalità delle curve algebriche tracciate sopra una superficie algebrica" (*Mathematische Annalen*, volume 62).

4. "Il teorema d'Abel sulle superficie algebriche" (*Annali di Matematica*, series 3, volume 12).

5. "Intorno al teorema d'Abel sulle superficie algebriche ed alle riduzioni a forma normale degl' integrali di Picard" (*Rendiconti di Palermo*, volume 21).

6. "Sul teorema di Riemann-Roch e sulle serie continue di curve appartenenti ad una superficie algebrica" (*Atti di Torino*, volume 40).

7. "Sulle curve algebriche virtuali appartenenti ad una superficie algebrica" (*Rendiconti del R. Istituto Lombardo*, series 2, volume 40).

8. "Osservazioni varie di geometria sopra una superficie algebrica o sopra una varietà" (*Atti del R. Istituto Veneto*, volume 65).

After the reading of the report the Guccia Medal was presented to Professor Severi.

Professor Mittag-Leffler then read his address "Sur la représentation arithmétique des fonctions analytiques générales d'une variable complexe."

He began by recalling that the central point of the theory of analytic functions is, with Weierstrass, the power series

$$p(x) = c_0 + c_1x + c_2x^2 + \cdots$$

and that this series is the source from which flows, successively and entirely by transformation and continuation, the analytic function in its entirety. He recalled the theorem of Weierstrass: If an analytic relation, however general or however special, exists between several different power series or their derivatives, this same relation subsists also for the functions in their totality.

The address had for its object to express the principal solutions, obtained during the last ten years, of the following problem: "To form arithmetic expressions of a variable x and of an infinite sequence of constants $c_0, c_1, c_2, \cdots$ which are linear in these constants and have the property of representing the given function $F(x)$, in a domain in which the $c_0, c_1, c_2 \cdots$, once determined, are defined in a unique manner." The first efforts toward the solution of this problem consisted in the investigations of the expressions which represent $F(x)$ not only within the circle of convergence C of $p(x)$ but also on the circumference of C at those points at which $F(x)$ is regular. Borel is the first who arrived at a solution more general, in having obtained an expression valid within a domain B which surrounds, in general, the domain C. Borel's idea that he had obtained, by his summation expression, the power series itself in the case where it diverges, was characterized as a play upon words, all the more intemperate since it gives rise to the illusion, entirely false, that he has been able to extend the limits of the theory of analytic functions beyond those fixed by the classic theory. As one is able, since the expression of Borel is convergent, to perform the same operation on the divergent series as on the convergent series $p(x)$, this scheme implies only the translation, for this special case, of the theorem of Weierstrass just stated.

The complete solution of the problem of the lecture has been *obtained finally and in several different ways since the new* conception of "the star" (of convergence) was first introduced by Mittag-Leffler in 1898, and one of the solutions, which was obtained by the generalizations of the integrals of Laplace, is attached to the important study of the functions $E_a(x)$ of Mittag-Leffler, as well as to that of the increase of the integral functions in an angle or along different semi-lines.

The lecturer closed by recalling the remarkable property of the function

$$E(x)=\int_{\delta} e^{e^{z}}\cdot e^{-e^{e^{z}}}\frac{dz}{z-x},$$

which tends indefinitely and uniformly toward zero when the variable x increases above any limit in the interior of a domain surrounding an angle, however small it may be, and which embraces the infinite part of the real positive axis, but which has the unexpected property of tending uniformly toward zero when the variable increases toward infinity along the real positive axis.

After a short recess Professor Forsyth read his address: "On the present condition of partial differential equations as regards formal integration."

The aim of the lecturer was to call attention to some of what may be called the classic methods of the past, whether recent or more remote, stating that in his opinion their usefulness had not been exhausted. He pointed out the need of a systematic treatment of the subject, since the theory remains fragmentary in form, indeed so much so that each individual success toward the solution of the problem is built up almost independently of its predecessors. The lecture was limited to the consideration of equations of the second order containing one dependent and two independent variables.

Two definitions of general integral were discussed, viz., that of Ampère and that of Darboux. The opinion was expressed that, until the old methods of integration are extended or new and more comprehensive methods given, we are in no position to declare what a comprehensive integral is. The lecturer then proceeded to what he considered the three principal methods of constructing the integral of a partial differential equation of the second order. These methods are the following:

(1) The method of Laplace : It is applicable to strictly linear equations and is effective for them solely when there are two independent variables, but the extension to equations of higher order still remains to be made.

(2) The method of Ampère: The results achieved by Ampère, particularly when regard is paid to the date, were on a

grand scale. Even now his ideas have not received their full development and his discriminating classification remains unamplified to this day. Unfortunately his method, while general in form and spirit, has no compelling force ; success in actual practice depends upon individual skill. His first class of equations, consisting of those whose integrals are expressible in finite terms without partial quadratures, remains as he defined it.

(3) The method of Darboux : When stated in full it can be effectually applied to equations which possess an intermediate integral. The general idea is based on the construction of another equation (still better two other equations) of the same order as the given equation and compatible with it. Moreover the method is progressive, that is to say, when the analysis shows that no compatible equations of the same order exist, similar analysis may lead to the construction of compatible equations of higher order.

One important gap still remains to be filled in this last method. It is true that when the equation is of the required character, the primitive can be constructed by this method, but no method has yet been devised for showing whether a given equation is of the required character. What is needed to fill the gap is one of two things, either some test which will show a priori the lowest order of compatible equations that can be associated with a given equation or else the construction of all the partial equations amenable to the method.

It happens only too often that an equation is not amenable to any one of the methods which have been mentioned. In that case the only plan of deriving information concerning the primitive is to have recourse to Cauchy's existence theorem. Usually this presents itself in inconvenient form, but at present we can do no better.

Some suggestive results have been obtained by proceeding from a different initial standpoint, using once more such classic processes as those of Euler and Lagrange. The principle of such methods is to regard the primitive as the origin and the partial equation as the consequence of the relation between the primitive and the equation.

We have thus seen that in this subject we can take only a few steps before reaching the boundary of present knowledge.

On Tuesday morning the four sections met for the first time, organized, and proceeded to the reading of special papers.

Each morning for the remainder of the week was devoted to the sectional meetings. On Tuesday afternoon two general papers were read. The first was by Professor Darboux on "Les méthodes et les problèmes de la géométrie infinitésimale."

The lecturer first gave a short account of the origin of infinitesimal geometry. Like other branches of human knowledge it had its origin in a practical problem, viz., that of mapping the surface of the earth. Lambert was the first to propose the problem, in full generality, of mapping the surface of the earth on a plane in such a way that infinitesimal elements shall be similar. It gave rise to some beautiful researches of Lambert, Euler, and Lagrange, but the complete solution was first given by Gauss in 1822. This was followed in 1827 by his Disquisitiones. In this last are found many ideas destined to great development in the modern theory of surfaces.

Professor Darboux next spoke of the activity in France during the same period and made special mention of the works of Monge, Dupin, Chasles, and Lamé, and the influence which they had on the development of infinitesimal geometry. Mention was also made of the geometers who followed these, and Ossian Bonnet was characterized as one to whom his works give almost the role of creator.

Having thus briefly sketched the history of the subject, the lecturer proceeded to what he considered the proper method of infinitesimal geometry. He said the method should be that of analysis which makes use of coordinate axes. The research should always be vivified and inspired by the geometric spirit. The method should not be followed blindly, for this being a grand highway will lead the more securely to the end desired, but the other roads of travel indeed have their charm. He advised the complete and frank introduction of imaginaries.

He then passed to the discussion of some of the unsolved problems in infinitesimal geometry.

1. The curves of constant torsion are of great importance in the study of surfaces applicable to a paraboloid of revolution, as these surfaces can be derived by a geometric construction from imaginary curves of constant torsion. Now it would be a problem worth the effort to determine all those curves of constant torsion which are algebraic or even unicursal. Attempts have been made at the problem, but a general solution is yet to come.

2. Bour announced that by applying the celebrated method of

Lagrange, the variation of constants, he had obtained all the surfaces applicable to any surface of revolution whatever, and in particular to a sphere. The manuscript was lost in a fire. Many geometers have sought to find the solution of Bour, but as yet no one has succeeded.

3. The initial conditions selected in the discussion of the problem of Cauchy should be as general as possible. For example, the following are suggested : If it is a question of an equation with two independent variables, the surface should be restricted to pass through a given curve when the equation is of the first order, but if the equation is of the second order the surface should be tangent the whole length of a curve to a developable surface. This method should lead in a precise manner to the notion of characteristics.

4. The above problem has been solved completely for minimal surfaces, but it is not to be confounded with the very different problem which has for its object the determination of a continuous minimal surface which passes through a given closed curve. Much has been done, but much still remains to be done with this latter problem.

5. Along with the problems of mapping, the problem of Tchebychef, somewhat generalized, was considered. Imagine a net, of any form and dimensions, composed of two series of threads attached firmly at their points of intersection so that the angles but not the sides of the meshes may vary. Suppose a solid bounded by any surface whatever be dropped into the net. Determine the form which the net assumes. In case the solid is spherical the solution can be connected with the determination of surfaces of constant curvature.

6. Some of the first and most elementary problems connected with the quadratic differential forms are yet to be solved. Here the great discoveries of Lie on the theory of groups have found and will continue to find beautiful applications.

7. The problem of the deformation of surfaces has been solved for the paraboloid, quadrics of revolution, and quadrics tangent once to the circle at infinity. Can the results be extended to quadrics in general? The answer, whether positive or negative, will be interesting.

The lecture closed with showing the important part which infinitesimal geometry plays in the study of partial differential equations.

Following Professor Darboux, Professor von Dyck presented a report: "Über die mathematische Encyklopädie."

On Wednesday afternoon Professor Newcomb read his address: "La théorie du movement de la lune: son histoire et son état actuel."

Since the law of gravitation was enunciated it has been the work of many mathematicians to compute theoretically the movement of planets, and in nearly all cases the theoretic movement agrees with the observed movement, but there are two noted exceptions, viz., Mercury and the moon. In the case of Mercury the difference can be accounted for by the presence of an unknown body between Mercury and the sun, but in the case of the moon we are confronted by an enigma. The importance of this divergence can be recognized after we have sketched the present state of the theory of the movement of the moon.

The law of gravitation is expressed by means of three differential equations, with time as the independent variable. The deductions are obtained by integrating these equations. The integrals give the coordinates of the body in terms of t and six arbitrary constants. The general solution in the case of the moon is not possible, but on account of the smallness of certain elements we can obtain a solution in the form of an infinite series arranged in powers and products of these elements. The coefficients are periodic functions of the time and we have solved the problem when we have calculated these. The method of Delaunay for handling these equations was discussed. He reduces the solution to the execution of a series of algebraic operations, comprising substitutions repeated without end but always approaching more and more nearly the exact value of the variables. This series for the coefficients, however, is so slowly convergent that it is not sufficient for the needs of modern astronomy.

There is a method used by Hansen which consists in substituting the numerical values for the constants, but since the derivatives cannot be calculated we must consider this method as unsatisfactory.

However there is a method which, in the speaker's opinion, has all the exactness demanded by the astronomy of our time. This is the method initiated by Euler and completed by G. W. Hill nearly a century later. Considering the mean movement of the moon about the earth and of the earth about the sun as known, the coordinates can be expressed in a series arranged according to powers of e, e' (the eccentricities of the two orbits), and $\gamma = \sin \frac{1}{2}I$. Then, y being such a coordinate,

$$y = P_0 + eP_1 + e'P_2 + \gamma P_3 + e\gamma P_4 + \cdots,$$

where the P's have the form

$$P = \sum {}^{\sin}_{\cos} (A + Bt).$$

Euler sought to develop separately the values of P, but found difficulty in introducing e, e', γ in B. Hill succeeded in developing P_0 in terms of the mean movement. He also gave a method for treating that part of the motion in perigee, independent of e, e', γ. E. W. Brown has attacked the problem also and has conquered its difficulties. The degree of exactness of Brown's work is all that could be desired.

These investigations all assumed that the sun is the only source of perturbation, but the planets also exert a small force which causes an acceleration of the mean movement of the moon, and there is besides a disagreement between theory and observation of which William Ferrel has found the partial cause in the effect of the moon upon the tides. But even this does not account for the entire disagreement. To explain the variations in the mean movement of the moon by the action of the tides, it would be necessary to suppose variations of almost a minute in our measure of time during the last two centuries. But the transits of Mercury seem to show that the variations cannot be more than a few seconds.

The lecturer closed by saying that the enigma seemed to him one of the most important and most interesting problems in celestial mechanics.

After a short recess Professor Lorentz gave his address: "Le partage de l'energie entre la matière ponderable et l'éther."

Kirchhoff reached the conclusion that the energy of radiation which exists in a unit volume of ether, so far as it depends upon a wave length comprised between the limits λ and $\lambda + d\lambda$, can be represented by an expression of the form

$$F(\lambda, \tau)d\lambda,$$

where F is independent of the special properties of the body. The lecture was especially concerned with the form of the function F. Wien has given to the function the form

$$F(\lambda, \tau) = \frac{1}{\lambda^5} f(\lambda\tau),$$

where instead of two independent variables λ, τ we have only the product $\lambda\tau$, but the form of f is still undetermined.

Professor Lorentz pointed out that Maxwell and Boltzmann had made use of two important branches of mathematics, viz., the calculus of probability and geometry of n dimensions. After showing the importance of these, he proceeded to limit the problem to considering a small mass M, composed of innumerable atoms animated and in motion in a rectangular box; further there are some charged particles or electrons free or held captive in the interior of the atoms. These electrons take part in the calorific movement of the atoms and should be regarded as the source of radiation.

The method used to treat the problem was that due to Gibbs. The state of the ether when some movable electrons are present is expressed by a system of partial differential equations seemingly very different from those of Hamilton, but which can be reduced to those by first establishing a theorem analogous to that of least action.

It is found that one meets a difficulty in applying the Gibbs method because the number of coordinates which define the electric field in the ether is infinite. It is necessary, therefore, to replace this system by a virtual system which has n degrees of freedom and treat the real system as the limit of this as n becomes infinite.

Finally the form of the function of radiation is found to be

$$F(\lambda, \tau) = \frac{16\pi\alpha\tau}{\lambda^4}.$$

This, however, is only valid for great wave lengths, but is definite enough for these. α is a universal constant which can be computed by making measurements on the infra-red rays. Planck by an entirely different method has obtained the formula

$$F(\lambda, \tau) = \frac{8\pi ch}{\lambda^5} \cdot \frac{1}{e^{3ch/2\alpha\lambda\tau} - 1};$$

which for long wave lengths agrees with the previous result.

The fact that F is independent of the special properties of the body is accounted for by the energy of agitation of the constituents, represented by $\alpha\tau$, which determines the intensity of radiation in the ether.

The lecturer closed by considering the merits and defects of the various methods for determining F.

On Wednesday evening a reception was given in the museum of sculptors at the Capitol, by the municipality of Rome. The halls were brilliantly lighted and beautifully decorated and their capacity, though very large, was tested. Mayor Nathan and other officials of the city were present. This reception was no less a delight to the members than the one on Sunday evening and the halls were not cleared until after midnight.

The usual program was followed Thursday morning, but in the afternoon the general addresses were suspended, and the congress adjourned to the Palatine, where a most delightful season was spent in viewing the many things of interest contained there. Guides who spoke different languages were furnished. After spending some time in inspecting the old Roman remains we were conducted to a beautiful spot on the Hill where a fine lunch was served. Everyone went away with the feeling that these non-sessions were among the most delightful parts of the congress.

Thursday evening all were provided with tickets to the concert at the Amphitheatre Corea. Those who loved music certainly enjoyed a treat and the others did not regret going.

Friday afternoon, Professor Poincaré being ill, his address was read by Professor Darboux. The title was: "L'avenir des mathématiques."

The author first examined the tendencies of the past and the present in order to be able to predict the future from these. He finds that there are two forces which influence mathematical thought. The first is the demand which physicists and engineers make on the mathematician for the solution of their problems. The second is the demand for the economy of thought. This economy is effected by organization and in many other ways; even the elegance to which mathematicians attach so much importance is attributed to this economy of thought.

In this economy it is not sufficient to give models to be followed. It is necessary that once knowing a given piece of reasoning, it can be repeated, in substance, in a few words. That is to say, a special choice of words is necessary. This choice of words has also another merit, it enables one to see that demonstrations made concerning one class of objects are valid for another also. Two good examples of this are the words group and invariant.

Having determined that these are the forces which influence mathematical thought to-day, the author proceeded to make predictions concerning various branches of mathematics.

Arithmetic: The development of arithmetic is now much behind that of algebra and analysis, mainly because of the discontinuity of its elements. The future of arithmetic therefore lies in the direction of making use of the advances in algebra and analysis in order to develop itself. For example, the parallelism between algebraic equations and congruences is almost perfect. Certainly this can be carried to completion.

The theory of primes in arithmetic seems to lack unity almost entirely, but without doubt unity will be produced by the consideration of a family of transcendental functions which permit, by the study of their singular points and the application of the methods of Darboux, the calculation asymptotically of certain functions in very great number.

Algebra: In the study of algebraic equations there still remains the problem of a system of invariants which do not change sign when the number of roots remains the same.

If one forms power series representing functions which admit the roots of an algebraic equation for singular points, the coefficients of the terms of higher order will furnish one of the roots with an approximation more or less close. Here is the germ of a process for numerical calculation of which a systematic study could be made.

The theorem of Gordan in the theory of invariants is yet to be extended.

Differential equations: In studying the functions defined by a differential equation we will not be satisfied until we have found groups of transformations which play the same rôle with respect to the differential equation that the groups of birational transformations play with respect to algebraic curves.

The lecturer emphasized the qualitative discussion of the curves defined by a differential equation. This is analogous to the investigation of the number of roots of an algebraic equation.

Partial differential equations: Here the idea of Fredholm can be applied to all the equations of the linear form. This remains to be carried to its completion.

Theory of functions: Here the theory of functions of more than one variable will occupy the attention; a mere analogy to functions of one variable is not sufficient. We must search for the things which will throw light on the difference between the theory for one variable and for several. For example, what is to take the place of conformal representation?

Theory of groups: The theory of continuous groups being much more advanced than the theory of Galois groups, it remains for the latter to make use of the analogies with the former.

Geometry: Analysis situs has been of great aid to mathematicians and still awaits to be completely constructed for space of more than three dimensions. When this is done we shall have an instrument which shall permit us really to see in hyperspace. The study of groups in geometry has contributed much to its growth. The study of groups of curves on a surface, analogous to the groups of points on a curve studied by Brill and Noether, has never been worked out.

Postulates: It seems at first that this field is well limited and that there is nothing to do when the inventory is completed. But when all are enumerated, there will be different ways of classifying them and each new classification will be of interest to the philosopher.

After a short recess Professor Picard gave his address: "L'analyse dans ses rapports avec la physique mathématique."

The principal object of the lecture was to show the mutual relation between physics and mathematics, that is to show that each derived much benefit from the other.

In the seventeenth century the development of kinetics and dynamics gave birth to the greatest advances in analysis; from that time dates the real beginning of modern analysis, it really came from mechanics. The origin of the notion of derivative is involved in the idea of the movability of things and of the rapidity with which phenomena occur. It was a decisive epoch in the history of mathematics when the development of mechanics conducted to the postulation that infinitesimal changes depend uniquely on the actual state of the system.

In the eighteenth century the development of mathematics in its most essential points was identified with that of mechanics. Clairaut in his study of the form of the earth considered, for the first time, curvilinear coordinates, d'Alembert used the two fundamental equations of the theory of functions and saw the significance of $\sqrt{-1}$. We find also functions of a complex variable considered by Euler and Lagrange.

The theory of potential and the analytic theory of heat have contributed much to mathematics. In these we often find also suggestions of the method of solution of the mathematical problem. Thus in this case physics has rendered a double service to mathematics.

There are also many examples of the service which mathematics has rendered to physics. The monumental work of Green on the applications of analysis to electricity and magnetism is an excellent example. In the study of the theory of waves we find also that differential equations have rendered much service; for example, in the theory of heat they showed that any variation is felt instantly in all directions and that therefore we cannot speak of the velocity of propagation. The influence which formal mathematics has had on physics and mechanics are instanced also by the equations of Laplace and the principle of least action.

The knowledge of the integrals of partial differential equations has caused some noted paradoxes in physics to vanish. The modern theory of functions seems at first to present but little of interest to the physicist, but extended study shows the theory of analytic functions to be of extreme importance to the mathematical physicist. The questions of domain of existence and of the prolongation of integrals have no less interest for the physicist than for the analyst. Indeed it seems to us that analysis is an indispensable instrument to the physicist and in some cases even a precious guide.

On Friday evening Professor Störmer gave his address: "Sur les trajectoires des corpuscles électrisés dans le champ d'un aimant élémentaire, avec application aux aurores boréales." The lecture was illustrated by a set of lantern slides. It was a résumé of a large memoir on the same subject, published in 1907 in the *Archives* of Geneva.

Partly by analysis and partly by the powerful methods of numerical and graphic integration of differential equations, Professor Störmer has succeeded in finding the properties of the trajectories in question. The numerical calculation represents a labor of nearly 5,000 hours. The author showed afterwards how the results found sufficed to explain a whole series of details in the remarkable experiments of M. Birkeland, in which he exposed a magnetic globe to a pencil of cathode rays, as explaining the principal character of the aurora borealis, for example the zones of auroras, the appearance in the night, the auroral rays and the remarkable phenomena of draperies of auroras.

The results of the applied analysis now make very probable the hypothesis of M. Birkeland, according to which the auroras are due to cathode rays, or something analogous, emitted by sun spots and meeting the atmosphere of the earth under the action of terrestrial magnetism.

On Saturday afternoon, Professor Veronese being ill, his lecture on "Geometria non-archimedea" was not read, but it will appear in full in the Atti del Congresso. The congress proceeded to the consideration of recommendations made by the various sections. The following resolutions were adopted :

The congress, having recognized the importance of an accurate examination of the program and the methods of instruction in mathematics in the secondary schools of the various nations, appoints Professors Klein, Greenhill, and Fehr as an international committee to study the question and to report upon it at the next congress.

"Section III (mechanics), after an exchange of views in which the importance of a unification of the notation of vectors was recognized, proposes to the congress the nomination of an international committee for the study of this question. The president of this section for the session of April 11 proposes to the congress to adopt its committee of organization to constitute this commission, and submits the list of names."

"The congress votes that at the next congress the constitution of the International Association of Mathematicians be presented."

"It is clear, from the exchange of views in Section III-b, that it would be highly desirable to effect a closer union between those who are occupied in perfecting mathematical methods and those who make practical applications. To this end the section urges that mathematics applied to the science of engineering be the object of a special session at the next congress. Section III-b further proposes the appointment of an international commission, which shall have charge of the work of this new section. The composition of this international commission shall be fixed by the bureau of the fourth congress."

"The Fourth International Congress of Mathematicians in Rome considers as a matter of maximum importance to the mathematical sciences, both pure and applied, the publication of the complete works of Euler. The congress greets with approval the initiative taken by the society of Swiss naturalists, and votes that the great work be taken up by the society itself with the help of the mathematicians of other nations. The congress urges the international association of the academies, and especially the Academies of Berlin and St. Petersburg, of which Euler was a celebrated member, to aid the enterprise."

The congress unanimously accepted the invitation of the Cambridge Philosophical Society to hold its fifth meeting at Cambridge in 1912.

Professor Mittag-Leffler, on the part of the Swedish mathematicians and the King of Sweden, invited the Congress to hold its sixth meeting at Stockholm in 1916. This of course could not be voted upon at this meeting.

Professor Darboux in the name of all the members of the congress thanked the committee and all those who had had a part in making the fourth international congress so important and so pleasant. The president then closed the congress officially.

While the official end of the congress was Saturday, there was still one more very pleasant non-session to be held on Sunday. All were furnished with tickets to Hadrian's Villa and Tivoli. The first stop was at Hadrian's Villa. Carriages were ready to take those who did not care to walk from the station to the villa. On entering the ruins, we found refreshments, provided by the municipality of Tivoli, ready and waiting to be served. After spending about two hours here, we proceeded to Tivoli, where a banquet awaited us. The banquet closed with toasts in Italian, French, German and Latin.

The afternoon was spent in visiting the cascades and the Villa d'Este. The returning trains arrived in Rome about 8 P. M. This was the unofficial but real close of the congress.*

C. L. E. MOORE.

ROME, *May*, 1908.

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* A report of the sectional meetings of the Congress will appear in the October BULLETIN.