THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE one hundred and forty-third regular meeting of the Society was held in New York City on Saturday, April 24, 1909, extending through a morning and an afternoon session. About thirty-five persons were in attendance, including the following twenty-six members of the Society:

Professor Maxime Bôcher, Professor A. S. Chessin, Miss E. M. Coddington, Professor F. N. Cole, Dr. E. B. Cowley, Dr. F. J. Dohmen, Professor L. P. Eisenhart, Professor W. H. Jackson, Mr. S. A. Joffe, Professor Edward Kasner, Mr. W. C. Krathwohl, Professor G. H. Ling, Dr. E. N. Martin, Professor G. D. Olds, Professor W. F. Osgood, Mr. H. W. Reddick, Professor D. E. Smith, Professor P. F. Smith, Dr. W. M. Strong, Mr. J. S. Thompson, Professor H. W. Tyler, Professor Oswald Veblen, Mr. H. E. Webb, Miss M. E. Wells, Professor H. S. White, Professor J. E. Wright.

The President of the Society, Professor Maxime Bôcher, occupied the chair, being relieved at the afternoon session by Vice-President Professor Edward Kasner. The Council announced the election of the following persons to membership in the Society: Miss W. B. Bauer, High School, Topeka, Kan.; Mr. W. W. Denton, University of Illinois; Mr. Meyer Gaba, University of Kansas; Professor W. H. Garrett, Baker University; Miss S. E. Graham, High School, Topeka, Kan.; Mr. G. F. Gundelfinger, Sheffield Scientific School; Professor W. A. Harshbarger, Washburn College; Mr. L. T. W. Hogrefe, Milwaukee, Wis.; Professor L. A. Howland, Wesleyan University; Mr. George Melcher, State Normal School, Springfield, Mo. Three applications for membership in the Society were received.

Professor H. S. White was reelected a member of the Editorial Committee of the Transactions for a term of three years beginning October 1, 1909.

The following papers were read at this meeting:

(1) Professor R. D. CARMICHAEL: "On certain functional equations."

(2) Professor R. D. CARMICHAEL: "Note on some polynomials related to Legendre's coefficients."

(3) Professor W. H. JACKSON: "A theorem concerning simple continued fractions."

(4) Professor W. H. JACKSON: "Shadow rails."

(5) Professor L. P. EISENHART: "The twelve surfaces of Darboux and the transformation of Moutard."

(6) Professor W. F. OSGOOD: "On Cantor's theorem concerning the coefficients of a convergent trigonometric series, with generalizations."

(7) Mr. L. S. DEDERICK: "Certain singularities of transformations of two real variables."

(8) Professor PAUL SAUREL: "On Fredholm's equation."

(9) Professor J. E. WRIGHT: "On abelian functions of genus 3."

(10) Dr. J. C. MOREHEAD: "A simplification of Lagrange's method for the solution of numerical equations (second paper)."

(11) Professor A. S. CHESSIN: "On gyroscopic couples."

(12) Professor EDWARD KASNER: "The interpretation of differential equations in line coordinates."

(13) Professor E. D. ROE, Jr.: "On the extension of the exponential theorem."

Mr. Dederick's paper was presented to the Society through Professor Bouton. In the absence of the authors Mr. Dederick's paper was read by Professor Bôcher, and the papers of Professors Carmichael, Saurel, Chessin, Kasner, and Roe, and Dr. Morehead were read by title. Professor Saurel's paper appears in full in the present number of the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In this paper Professor Carmichael considers the equations

$$h(x+y)\,h(x-y) = [h(x)]^2 + [h(y)]^2 - c^2,$$

$$g(x+y)\,g(x-y) = [g(x)]^2 - [g(y)]^2.$$

He finds that the general solution of the first is

$$h(x) = \pm \tfrac{1}{2}c(e^{kx} + e^{-kx})$$

and shows that, except for a constant factor, several related functional equations have the same solution. The second of the given equations has two solutions

$$g(x) = \pm \frac{g'(0)}{2k}(e^{kx} - e^{-kx}), \quad \text{if} \quad k \neq 0,$$

$$g(x) = \pm g'(0)\ x, \quad \text{if} \quad k = 0,$$

where $g'(0)$ is an arbitrary constant.

2. In his second paper Professor Carmichael considers a class of functions

$$P_{n\alpha} = \frac{1}{2^n n\,!}\,\frac{d^n}{dx^n}(x^{2\alpha} - 1)^n,$$

and shows that several properties of Legendre's polynomials are properties also of this generalized function. P_{n1} is the usual Legendre coefficient.

3. In his first paper Professor Jackson considers all those integral multiples of a given number for which the fractional part lies between given limits. It is shown that the integral multipliers satisfying this condition possess a periodic formation, one period within another, but that these periods are subject to breaks at calculable intervals.

4. In his second paper Professor Jackson applies the results of his first to calculate the position of the shadow rails observed when one set of rails is viewed through another. It is established that the spacing of the shadow bands depends only on the spacing of the two sets of rails and not on their breadths, and that when the position of the observer is altered the shadow bands move with magnified velocity in the same direction as that of the apparent motion of the more closely spaced set of rails relative to the other set.

5. The problem of the infinitesimal deformation of a surface S is equivalent to the determination of a surface S_1 corresponding to S with orthogonality of linear elements, or of an associate surface S_0 whose tangent plane is parallel to the corresponding tangent plane to S and whose asymptotic lines correspond to a conjugate system on S. In the fourth volume of his Leçons Darboux shows that nine other surfaces are associated with S, S_1, S_0 in such a way that any three successive surfaces of the closed cycle are in relations similar to those which hold between S, S_1, and S_0. Darboux is led to the result that the cycle is

closed after he has calculated the coordinates of the various surfaces. Professor Eisenhart, in the first part of his paper, is concerned with the reasons for this closure and with an investigation as to the possibility that all of the twelve surfaces be not distinct. In the second part of his paper, he considers the group of surfaces which figure in the infinitesimal deformations of a surface S and the three surfaces S', S'', S''' which arise from S by transformations of Moutard,* these surfaces being such that the pairs S, S'; S, S''; S', S'''; S'', S''' are the focal surfaces of W-congruences. In general there are sixteen distinct surfaces in the group. An exceptional case of importance is that for which the two surfaces associate to S are associate to one another.

6. Professor Osgood's paper contains a simple proof of the theorem that if a trigonometric series

$$\Sigma(a_n \cos nx + b_n \sin nx)$$

converges for all values of x in a certain interval, however short, its coefficients approach zero as their limit. The theorem is generalized to series of the form $\Sigma A_n f_n(x)$, where f is continuous in an interval (a, b) and in every subinterval (α, β) takes on values numerically greater than a certain positive constant l at some point of the interval, provided $n \geqq m$. If such a series converges in (a, b), then

$$\lim_{n=\infty} A_n = 0.$$

Further generalizations to multiple series

$$\Sigma f_{m,n}(x, y), \quad \Sigma f_{m,n,p}(x, y, z),$$

are given.

The paper will be offered for publication in the *Transactions*.

7. The object of Mr. Dederick's paper is to discuss general properties of those singularities of transformations of two real variables $u = \phi(x, y)$, $v = \psi(x, y)$ which are due not to discontinuities in the functions ϕ or ψ, or their partial derivatives, but to the vanishing of the Jacobian determinant $J = \phi_x\psi_y - \phi_y\psi_x$. The most important results are the derivation and use of a general formula for the transformation of the derivative of one variable with respect to the other, which applies with an unimportant exception to all singularities of

* Bianchi, Lezioni, vol. 2, p. 69.

this sort, and the classification of such singularities on the basis of this formula. The transformed second derivative is in general infinite except in the simplest case. Some geometric properties discussed are those of the critical slopes, the preservation of regularity of an arc, the criterion for the transformation of a curve into a point, and the order of contact of the curves $\phi = \text{const.}$ and $\psi = \text{const.}$ with each other and with $J = 0$. A reduction of the transformation to the normal form, $u_1 = x_1$, $v_1 = y_1^2$, is obtained for any neighborhood of an arc of $J = 0$ in which one of the expressions $\phi_x J_y - \phi_y J_x$ and $\psi_x J_y - \psi_y J_x$ remains different from zero, the reduction being effected by transformations of the two planes whose Jacobians nowhere vanish. The following theorem on implicit functions is derived: If $F(u, x)$ and all its partial derivatives of order less than n vanish at (u_0, x_0) and the partial derivatives of the nth order are continuous near (u_0, x_0) and not all zero at that point, then to every real simple root of the equation of the nth degree for formally determining du/dx, corresponds a solution $u = f(x)$ which has a continuous derivative near (u_0, x_0).

9. Professor Wright's paper is concerned with the determination of the differential equations satisfied by abelian θ functions of genus 3. The equations are obtained in covariantive form, and they lead to the generalization of Kummer's quartic surface. This turns out to be a three spread in space of six dimensions, of the 8th order in the hyperelliptic, of the 16th order in the non-hyperelliptic case.

10. In an earlier paper* Dr. Morehead showed how binomial synthetic division may be applied to the approximate expression of the incommensurable real roots of numerical equations in terms of simple continued fractions

$$a_1 + \frac{1}{a_2 +} \frac{1}{a_3 +} \cdots \frac{1}{+ a_n}.$$

This method is modified, in the present paper, so as to give the approximate expression of the roots in continued fractions of the form

$$a_1 + \frac{b_2}{a_2 +} \frac{b_3}{a_3 +} \cdots \frac{b_n}{+ a_n},$$

the b's being arbitrarily chosen.

* Read before the Chicago Section, April 9, 1909.

The similar process for the expression of the roots in ascending continued fractions

$$a_1 + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \cdots + \frac{a_n}{b_n}$$

is a generalization of Horner's method, as the above reduces to the decimal fraction $a_1\, a_2\, a_3 \cdots a_n$ for $b_2 = b_3 = \cdots = b_n = 10$.

11. The well known property of rapidly revolving bodies to resist efforts tending to deflect their principal axes is due to the rise of couples among which that of the so-called turning forces produces most of the effect. This couple is generally assumed to be equal to $C\omega\phi' \sin\theta$ where C, ω, ϕ, and θ denote respectively the moment of inertia about the axis of spin, the angular velocity of deflection, the angular velocity of spin S, the angle of inclination between the axis of spin and the axis of deflection. In the present paper Dr. Chessin shows that the correct expression of the turning couple is given by the following values of its projections for the three eulerian axes:

$$C\omega(\phi' + \omega\cos\theta)\sin\theta, \quad (2A - C)\omega\theta'\cos\theta, \quad C\omega\theta'\sin\theta,$$

which reduce to the commonly used approximate value $C\omega\phi'\sin\theta$ when $\theta' = 0$ and ω is very small compared to ϕ'.

12. An oriented straight line may be defined most simply by means of its Hessian coordinates (u, v), the latter representing the length of perpendicular dropped from the pole and the former representing the angle which this perpendicular makes with the initial line. A differential equation of the form $F(dv/du, u, v) = 0$ then defines a single infinity of oriented curves. The interpretation of derivatives is very simple. Professor Kasner discusses in particular the systems of curves represented by linear and Riccati equations. These types may be regarded as natural extensions of systems of parallel curves. In constrast with the usual statements for cartesian coordinates (x, y), the results are here invariant under the displacement group.

13. Professor Roe's paper contains an elaboration of the theory of the exponential function.

F. N. Cole,
Secretary.