ELEMENTARY MECHANICS.

A First Statics. By C. S. JACKSON, M.A., R. M. Academy, Woolwich; and R. M. MILNE, M.A., R. N. College, Dartmouth. [Dent's Mathematical and Scientific Text-books, edited by W. J. Greenstreet.] London, J. M. Dent & Co., 1907. Crown 8vo. vii + 380 pp. 4 shillings.

A Text-book of Mechanics. By LOUIS A. MARTIN, JR., M.E., A.M., Stevens Institute of Technology. Vol. I, *Statics*, 1906. 12mo. xii + 142 pp. $1.25. Vol. II, *Kinematics and Kinetics*, 1907. 12mo. xiv + 214 pp. $1.50. New York, John Wiley and Sons.

PERHAPS it is immaterial to our colleges whether their average graduate knows even the simplest principles of physics, — so as to be able, for example, to explain the advantages of a simple set of pulleys, or the physical basis of time measurements, or the easy balancing of a moving bicycle, or when a leaning ladder could be safely ascended if slipping were opposed only by friction. At least, such an inference seems warranted so long as educators suppose that familiarity with records of human activities of various sorts is a compensation for ignorance of natural laws, and until they cease placing their labels of education upon persons who have not thoroughly studied even the broader sciences. The common and dense ignorance of one fundamental science could be removed by even a first course in mechanics (developed from the physical side), whether or not the average student should later elect the more mathematical study of this science, whose problems have so greatly stimulated the growth (and still defy the powers) of mathematical analysis. These two kinds of courses in mechanics are somewhat typified by the books under review.

The First Statics is not purely mathematical, as it employs numerous simple experiments; but it is far from being a laboratory manual. Experiments are suggested merely to introduce the laws — and the first ideas of mechanics should always come in that way — while deductive analysis is used throughout, though with no stress on rigor. Trigonometry is employed, but not calculus, even in finding centers of gravity.

The authors treat the lever and the equilibrium of parallel forces before introducing the parallelogram of forces, which follows next with graphical methods for three concurrent forces, and leads to analytical methods of composition and resolution, and the properties of couples. Several simple machines are then explained: weighing-bridge, "penny-in-the-slot" machines, tackle, differential pulley, Chinese wheel and axle, lifting crab, worm and wheel, and screw. There is a good treatment of friction and an extended discussion of centers of gravity — of a few bodies geometrically, of some others by applying the theorems of Pappus, and of any number of particles analytically by taking moments. Graphical analysis of framed structures, the use of the link polygon, and the general analytical conditions for the equilibrium of coplanar forces are treated at some length, followed by an exposition of the relative advantages of graphical and of analytical methods, and of the usefulness of the principle of work. The volume concludes with a chapter on forces in space, and a good set of miscellaneous problems.

Features of this very worthy text which deserve special commendation are the presence of occasional questions as to the probable extent and cause of error; the carefully graded treatment, with simplest beginnings; the excellent set of some 700 problems (especially good on taking moments); and the admirable treatment of the parallelogram of forces, not offering a pretended proof of it, but assuming it as the result of experience, after the student has been convinced by experiments. Indeed, important throughout is the inductive presentation of new ideas, whereby the student is prepared by experiment or special example for the statement of a principle. Of course, in the teaching of pure mathematics, this office of the experiment is not dispensed with, but filled by the numerical example. To illustrate: the average student of analytical geometry is puzzled if directly confronted with $S_1 + kS_2 = 0$ as the equation of a straight line through the intersection of the lines $S_1 = 0$ and $S_2 = 0$. But if he has first seen several numerical cases, and has observed how the new line changes for different values of the parameter, he can readily grasp the facts concerning the general equation, which should not even be written until the examples have been understood. Never should any new concept be introduced save through special cases. The soundness of this pedagogic principle is so incontestible that one marvels at the small part it plays in our texts.

But to return: there are parts of the First Statics which are open to criticism. In proving the theorems of Pappus for curves, would it not be better to employ exact (though elementary) limit statements than to say (page 224): "since the proposition holds for the sum of all the rectangles, it will hold for the area of the curve itself?" Or (page 228): "the solid figure formed . . . will closely coincide with the sphere. We therefore infer that the surface of the sphere itself is equal to $4\pi R^2$." It might be well in some proofs to point out what statements are not based on earlier principles but are merely assumed as true; thus it would be interesting to question whether step (1) in Stevin's solution (page 62, example 5) need be admitted unless perpetual motion is supposed impossible. It is somewhat careless to equate a pure number to a number of dimensional units, as (page 238) "$225/80 = 2.81$ feet;" or (page 302) "$125 \cos 40°/\sin 65° = 106$ lbs. nearly."* And such experiments as that on page 75 seem too clumsy to be useful.

The following misprints have been noted: page 76, example 33, for 30 read 31; page 114, top, for orignial read original; page 123, bottom, for enses read senses; page 187, bottom, for revolving read resolving; page 207, figure, for interior B read B'; page 285, frame diagram, for one b read b'; page 338, line 4, for e read be. The reviewer would also suggest more paragraph headings. American students should be informed: whether (page 144) a "piece of cotton" is a thread; (page 201) that the given weight of a penny is not that of a cent; and that an English "trapezium" (page 204 et seq.) is an American "trapezoid."

Professor Martin's Text-book of Mechanics presents, on the other hand, a purely mathematical treatment, using no calculus in the first volume but introducing it gradually in the second.

The Statics begins with the parallelogram of forces, assumed as an experimental datum, and treats the composition and resolution of forces, couples, conditions for equilibrium in two dimensions, and centers of gravity (rather briefly, without the theorems of Pappus). A summary of methods is followed by the consideration of various machines: lever, wheel and axle, systems of pulleys, inclined plane and wedge, bent lever balance, differential wheel and axle, and platform scales.

* The same criticism applies to the other text under review; e. g., Vol. I, p. 2; Vol. II, p. 19.

Except for an appendix on three-dimensional structures, and problems for review, the rest of the volume is given to the graphical methods used for centroids, resultants, and framed structures.

In kinematics the composition and resolution of velocities and accelerations in the motion of a particle are followed by space-time and velocity-time curves, the composition of simple harmonic motions, and a brief treatment of the translation and rotation of rigid bodies. The author begins kinetics with some explanatory comments on Newton's laws of motion, follows with a commendable digression on the theory of dimensions; and discusses the free translation of a particle or mass center for various forces, constrained motion, and the rotation of a rigid body, moments of inertia being introduced very naturally. He then treats work and energy, and the application of the principle of work to machines; and concludes with a chapter on impact, and a good set of review problems.

Evidences that the text is the work of a teacher appear throughout both volumes; as, for example, in the lucid explanation of the angle of repose, and in the facility with which practical problems are introduced early in the first volume; or, again, in the excellent method of analyzing very fully many typical problems (although one might wish to see more of these examples used to introduce the discussion of principles). With such a text the student must, in order to solve problems, gain an understanding of the subject by study of the solved problems and general theory; while some mathematical texts give him such explicit working rules that he can mechanically follow the rules without understanding the theory, with the result that, when the memorized rule is forgotten, he has nothing left. Still worse in the reviewer's opinion is the tendency of rules to deprive the student of the necessity for formulating methods independently; for one glaring fault of our educational system is that it cultivates so little originality and ability to think. Trained men are good, but educated men are better! Working rules have their place (along with tables) in practice, but in teaching they should be used with moderation.

There is of course, no protest against collecting the results of discussions, as the author has done in several places, thereby adding further to the success of his effort (mentioned in the preface) to make the book thoroughly teachable. In fact, the reviewer believes that as a text this work ranks with the best previous books, despite a number of criticisms which must be stated.

For one thing, the desirability of presenting at the outset all the material of the introduction, which precedes statics, may be questioned; but more serious is a logical incompleteness similar to that which pervaded our elementary geometries before the modern revision. For example, just as we used to "prove" theorems concerning the area of a circle, which had never been defined (unless by a meaningless phrase), so here (pages 36, 43), although neither the acceleration nor any component has been defined for curvilinear motion, the author proves that $\alpha = \sqrt{\alpha_x^2 + \alpha_y^2}$ and not d^2s/dt^2. Also (page 7) we find assertions concerning velocity when $\Delta s/\Delta t$ varies, although the word has been given a meaning only when that ratio is constant; and similar remarks apply to acceleration (page 11). It may easily confuse a student to read (page 38): "In curvilinear motion the velocity can even be constant and still there must be an acceleration," when his only definition of acceleration is (page 10): "the time-rate of change of velocity;" or (page 11), "acceleration is always the first derivative of velocity with respect to time."

Again from the mathematical standpoint it is unsatisfactory to have a body treated as consisting of a finite number of particles, without an adequate transition from the sign of summation to that of integration. Also unfortunate though less important is the use of three steps (page 13) to conclude that if $v = ds/dt$ and $\alpha = dv/dt$, then $\alpha = d^2s/dt^2$; and do the remarks (page 8) about t being "an equicrescent variable so that dt is constant" mean anything more than that we choose t as the independent variable and can therefore take dt constant if we wish? Will the remarks concerning a rolling wheel (page 62) be clear before the student has heard of the instantaneous center; and would it not be shorter and more natural (page 103) to let s denote the distance from the center of the earth? In the solved problem on page 18 (Volume I) the use of similar triangles could be replaced by trigonometric methods by merely lettering an angle. It is inaccurate to say (Volume I, page 46): "CA_1 can only equal CA_2 as $CA_2 \doteq \infty$;" or write as equal the alternative values for t_1 and for t_2 (Volume II, page 152). It might be well to state that F is regarded as constant (Volume II, page 71) in defining impulse, and (page 167) in defining work; and to insert (page 140, line 3) the words "to be proved" after are.

There are some typographical errors: Volume I, page 59, middle, for $\Sigma(Fd)$ read $-\Sigma(Fd)$; Volume II, page 33, omit to in sixth line from bottom; page 88, bottom, for ΣF read ΣF_x; page 101, line 2, insert a comma before e; page 165, figure, for ax and ay read a_x and a_y; page 189, line 7, for are read is.

Nevertheless, it must be said that each of the texts under review is well printed on the whole, and presents a very attractive appearance. And despite the extent of the criticisms here offered, both books should prove thoroughly practical, often suggestive and highly satisfactory. Neither text is indexed, but the deficiency is not serious in such a subject.

As these pages go to press, a second edition of Professor Martin's Mechanics is announced by the publishers. The early appearance of a second edition is perhaps the most substantial evidence of the success of the book.

F. L. GRIFFIN.

SHORTER NOTICES.

Opere Matematiche di Eugenio Beltrami. Pubblicate per cura della Facoltà di Scienze della R. Università di Roma. Milano, T. I, con ritratto e biografia dell' autore, 1902, vii + 437 pp.; T. II, 1904, 468 pp. 4to.

THESE two volumes contain 43 memoirs of Beltrami's, ranging from 1861 to 1873, and an epilogue, by Cremona which serves as an introduction. Among the papers included are those by which Beltrami earned his world-wide fame and established once for all his claim to be regarded as one of the founders of the modern subject of differential geometry. There are the memoirs on differential parameters, on the complex variable spread over an arbitrary surface, the memoirs on non-euclidean geometry, referring to ordinary and hyperspaces, the theory of geodesic lines, including the famous solution of the problem of geodesic representation of a given surface on the plane. The second volume contains also an elaborate study of the kinematics of a fluid.

If we except, perhaps, the last-named voluminous research, it may be said that the main results arrived at by Beltrami are now generally known. They have found their way into the more extensive text-books; so that we need not repeat what is, or may be, familiar to every student. Nevertheless the present