

There are some typographical errors: Volume I, page 59, middle, for $\Sigma(Fd)$ read $-\Sigma(Fd)$; Volume II, page 33, omit to in sixth line from bottom; page 88, bottom, for ΣF read ΣF_x ; page 101, line 2, insert a comma before e ; page 165, figure, for ax and ay read a_x and a_y ; page 189, line 7, for are read is.

Nevertheless, it must be said that each of the texts under review is well printed on the whole, and presents a very attractive appearance. And despite the extent of the criticisms here offered, both books should prove thoroughly practical, often suggestive and highly satisfactory. Neither text is indexed, but the deficiency is not serious in such a subject.

As these pages go to press, a second edition of Professor Martin's *Mechanics* is announced by the publishers. The early appearance of a second edition is perhaps the most substantial evidence of the success of the book.

F. L. GRIFFIN.

SHORTER NOTICES.

Opere Matematiche di Eugenio Beltrami. Pubblicate per cura della Facoltà di Scienze della R. Università di Roma. Milano, T. I, con ritratto e biografia dell' autore, 1902, vii + 437 pp.; T. II, 1904, 468 pp. 4to.

THESE two volumes contain 43 memoirs of Beltrami's, ranging from 1861 to 1873, and an epilogue, by Cremona which serves as an introduction. Among the papers included are those by which Beltrami earned his world-wide fame and established once for all his claim to be regarded as one of the founders of the modern subject of differential geometry. There are the memoirs on differential parameters, on the complex variable spread over an arbitrary surface, the memoirs on non-euclidean geometry, referring to ordinary and hyperspaces, the theory of geodesic lines, including the famous solution of the problem of geodesic representation of a given surface on the plane. The second volume contains also an elaborate study of the kinematics of a fluid.

If we except, perhaps, the last-named voluminous research, it may be said that the main results arrived at by Beltrami are now generally known. They have found their way into the more extensive text-books; so that we need not repeat what is, or may be, familiar to every student. Nevertheless the present

edition is highly to be welcomed. Through the necessary process of condensation an author's results often lose much of their freshness, so that the careful student will always be glad to revert to the original work. Thus the beautiful theorem concerning the deformation of an arbitrary surface to which a normal system of rays is rigidly connected comes out quite naturally where it stands (page 121 of the present edition, volume I), while the much shorter proof by which Beltrami's development has since been replaced has a somewhat artificial aspect, and fails to suggest how one may arrive at such a statement. Numerous theorems, to be found in text-books without any references, were probably discovered by Beltrami. At least Beltrami himself (who had a vast knowledge of the literature, foreign as well as Italian, and was most careful in bestowing due credit on others) apparently considered them as new. And there are many other interesting theorems still as fresh and new as they were when published for the first time.

A peculiar charm of these writings, not often to be found in work of high originality, lies in the simplicity of Beltrami's exposition. He was evidently anxious to meet the needs of his readers, and not to presuppose unnecessarily an amount of knowledge on the part of the student that in most cases, unfortunately, is not at hand. It is to be regretted that the mass of material by which nowadays all editors of mathematical journals are overwhelmed tends more and more to make this pleasant manner of writing unfeasible.

Of minor details we may point out one that has already been emphasized by Herr von Mangoldt. In one of Beltrami's earlier papers (volume I, page 75)—where, judging from its title, nobody would expect such a thing—the chief properties of a figure were studied which, fourteen years later (viz., 1879), was rediscovered and fully discussed by C. Stéphanos. The figure is now generally known under the name of desmic configuration, given to it by the latter eminent geometer. Beltrami seems also to have been the first to determine the apparent size of a surface of the second degree. His paper on this subject (the fourth in the present edition) is dated 1863, and so precedes by twenty years the corresponding publication of H. A. Schwarz (*Gesammelte Werke* II, page 312).

In Beltrami's work geometrical ideas are prominent throughout. His method, however, was exclusively that of analysis. It would seem that the so-called pure geometry did not attract

him much. He certainly saw the traps into which contemporaneous writers had fallen and was fully aware of the limited range of the synthetic method.

For further information we refer the reader to a sketch of Beltrami's life-work by E. Pascal, published in the *Mathematische Annalen* (volume 57, 1903, pages 65-107), and thereby made generally accessible.

The editors, who modestly disappear behind their work, have taken great pains that the edition of the writings of their illustrious compatriot should appear in a dignified form. It will, for example, be difficult to find misprints. The paper is excellent, as also is the printing, which was done in the Tipografia Matematica di Palermo.

E. STUDY.

Études sur les Angles imaginaires. Par GEORGES DE LAPLANCHE. Paris, A. Hermann, 1908. 8vo. 135 pp. 3 francs.

THIS volume devotes considerable space to the development of the ordinary formulas, the computations, and the construction of graphs of trigonometric functions of complex numbers. The imaginary angle spoken of is merely the complex argument of these trigonometric functions, and in no way is it connected with the geometric conception of angle. De Moivre's formula is made the basis of all the developments, with no attempt at rigorous proof, while the graphical treatment rests on the usual Argand diagram. A gross misstatement is made on page 35 and repeated on page 36, to the effect that in $\sinh x$ and $\cosh x$ (written $\text{sh } x$, $\text{ch } x$), defined thus

$$\sinh x = \frac{1}{2}(e^x - e^{-x}), \quad \cosh x = \frac{1}{2}(e^x + e^{-x}),$$

the argument x is the length of the arc of an equilateral hyperbola, measured from its vertex. The equations of this hyperbola may be written

$$X = \cosh x, \quad Y = \sinh x,$$

whence

$$s = \int_0^x \sqrt{1 + 2 \sinh^2 x} \, dx.$$

This is obviously not equal to x . The author seems to have