him much. He certainly saw the traps into which contemporaneous writers had fallen and was fully aware of the limited range of the synthetic method.

For further information we refer the reader to a sketch of Beltrami's life-work by E. Pascal, published in the *Mathematische Annalen* (volume 57, 1903, pages 65–107), and thereby made generally accessible.

The editors, who modestly disappear behind their work, have taken great pains that the edition of the writings of their illustrious compatriot should appear in a dignified form. It will, for example, be difficult to find misprints. The paper is excellent, as also is the printing, which was done in the Tipografia Matematica di Palermo.

E. STUDY.

Études sur les Angles imaginaires. Par GEORGES DE LAPLANCHE. Paris, A. Hermann, 1908. 8vo. 135 pp. 3 francs.

THIS volume devotes considerable space to the development of the ordinary formulas, the computations, and the construction of graphs of trigonometric functions of complex numbers. The imaginary angle spoken of is merely the complex argument of these trigonometric functions, and in no way is it connected with the geometric conception of angle. De Moivre's formula is made the basis of all the developments, with no attempt at rigorous proof, while the graphical treatment rests on the usual Argand diagram. A gross misstatement is made on page 35 and repeated on page 36, to the effect that in sinh x and cosh x (written sh x, ch x), defined thus

$$\sinh x = \tfrac{1}{2}(e^x - e^{-x}), \quad \cosh x = \tfrac{1}{2}(e^x + e^{-x}),$$

the argument x is the length of the arc of an equilateral hyperbola, measured from its vertex. The equations of this hyperbola may be written

$$X = \cosh x, \quad Y = \sinh x,$$

whence

$$s = \int_0^x \sqrt{1 + 2\sinh^2 x}\, dx.$$

This is obviously not equal to x. The author seems to have been misled in trying to extend the circular argument of cos x, sin x. This, it is true, may be taken as an arc, but it has long been known that in order to extend the analogy to the hyperbolic functions it is necessary to take as argument the ratio of the *sector* to one half the square of the radius. The baneful influence of considering trigonometric functions as lines depending on a linear argument is evidently not yet extinguished.

However, the author makes no use of his erroneous statement, but depends solely on series and De Moivre's formula, so that his formulas are correct enough.

The purpose of the work, scarcely realized in its treatment, is stated thus:

"Nous nous proposons de rechercher si le nombre imaginaire a des lignes trigonométriques: sinus, cosinus, tangentes, circulaires ou hyperboliques. . . . Nous en établissons la trigonométrie. Nous montrons de nouveaux moyens pour résoudre certains problèmes."

JAMES BYRNIE SHAW.

Vorlesungen über bestimmte Integrale und die Fourierschen Reihen. Von J. THOMAE. Leipzig, Teubner, 1908. 8vo. vi + 182 pp. 7.80 Marks.

This book undertakes to give a rather general view of the subjects mentioned in its title. It is somewhat more on the order of a descriptive course than either a systematic development or a practical handbook. Thus, in the first fifty-seven pages the student will see unfolded before him a view of the main theorems with some regard to the dangerous places near them. He will learn that there are such functions as Euler's $E(x)$, Dirichlet's function, Riemann's classic function (x), written here $r(x)$, Riemann's convergent series with an infinity of discontinuities

$$f(x) = r(x) + \frac{r(2x)}{2^2} + \frac{r(3x)}{3^2} + \cdots.$$

He will find that there is a definition of integral as the limit of a sum, which indeed is suggested by the inversion of a differentiation, and that under this definition many functions become integrable, for example those of Riemann mentioned above. The first mean value theorem he finds to hold equally for such functions, and the second mean value theorem is developed. He also finds that sometimes the variable may be changed,