geometry, lines in a plane, general plane curves, conics, cubics, general space geometry, algebraic surfaces, families of surfaces, quadrics, twisted curves; line geometry, transformations in a plane; differential geometry of the plane and of space; probabilities, errors, numerical calculations, graphics, vector analysis and quaternions. The other headings mentioned are equally fully treated. Many topics have a place assigned for future exposition. In a few years this small encyclopedia will be almost a necessity for student, teacher, and investigator.

The book is illustrated with a portrait of Lord Kelvin, and his biography opens the introduction. The properly "yearbook" topics are a calendar, astronomical data, lists of journals and proceedings and transactions of learned societies, new books, necrology for 1908, lists of teachers of mathematics and physics in Germany. The errors in the book are few, so far as the reviewer noticed in his reading; those existing are noticeable at once and doubtless will disappear in the next volume.

JAMES BYRNIE SHAW.

Statistique Mathématique. Par H. LAURENT. Paris, Octave Doin, 1908. vi + 272 + xii pp.

THE author states that, for him, the object of mathematical statistics is to indicate and investigate methods of making good observations, when the point in question is to make numerical estimates concerning matters which interest economists. He has thus limited his purposes to matters which relate to specific applications. This fact may account, in part, for the entire omission of that important body of mathematical statistics which has been developed in close connection with applications to biology. However, these methods have been applied by others to problems of economics.*

It is well stated in the preface that it is a very common error to suppose that those who direct statistical investigations do not need to know mathematics. The author remarks that official statistics are not good, in general, because those who direct statistical investigations are not prepared for the work, and that if it is not necessary to exact of the statistician that he have a command of universal science, it is necessary, at least, that he should have surveyed the field of scientific knowledge.

*See Yule, *Journal of the Royal Statistical Society*, vol. 60, pp. 812–854. Norton, Statistical Studies in the New York Money Market.

The book begins with the elements of probability — leading soon to the principle of Bayes on the probability of causes. In the proof of the inverse of the theorem of Bernoulli, it may be worth while to call attention to a few details which may confuse the reader: If l is to be used as indicated on page 28 as a deviation from a/s, instead of using l/s as on page 26 for this purpose, the expression $P = \theta(ls/\sqrt{2\alpha\beta s})$ on page 30 becomes $P = \theta(l\sqrt{s^3/2\alpha\beta})$. The expression $P = \theta(ls/\sqrt{2\alpha\beta s})$ is correct if l/s is used throughout as a deviation from a/s. On page 29, in the formula for P, the radical should be $\sqrt{s/2\alpha\beta\pi}$ instead of $\sqrt{\alpha\beta/2\pi s}$.

On page 37 there appears to be a numerical error such that we should read $\theta(l/628)$ instead of $\theta(l/6280)$ but the conclusion is all the more valid if this change is made.

In developing what he calls a general theory of errors, the author follows the well-known method of obtaining the normal curve from the hypothesis of Gauss in regard to the arithmetic mean. In presenting the method of least squares, the analysis of Laplace is followed rather closely, and the approximation consists in neglecting

$$\int_{-\infty}^{\infty} \phi(\epsilon)\,\epsilon^h d\epsilon \quad (h > 2),$$

where $\phi(\epsilon)d\epsilon$ is the probability (facilité) of an error between ϵ and $\epsilon + d\epsilon$, but no further assumption is made in regard to the form of the function $\phi(\epsilon)$. The integral

$$\frac{1}{\pi}\int_{-\infty}^{\infty} e^{\mu(t-a)\sqrt{-1}}\,\frac{\sin \mu l}{\mu}\,d\mu$$

is used as an auxiliary function to evaluate the integral which expresses the total probability of the concurrence of errors.

The greater portion of the book is given to applications rather than to pure theory. The various sources of statistics in France are indicated, and conditions are discussed which must be satisfied in order that an observation which is known only by description may be of value. Agricultural statistics which report crops of grain to seven significant figures are severely criticised in the following terms: "As there is no cause to suspect the honesty of those who publish these figures, it is necessary to infer their ignorance in scientific matters or their native simplicity, and then only natural to doubt the exactness of the first figures."

It is stated that of all statistics relative to man the best and perhaps the most important deal with the duration of human life and with immigration. The book contains the well known methods (aside from the method of moments) of graduating mortality statistics, with the somewhat reasonable à priori considerations which lead to the various types of functions used in such graduations.

An argument is given to establish that part of the law of Malthus

$$p = p_0 e^{k(t-t_0)}$$

which states that populations have a tendency to increase in geometrical progression, but of the part stating that subsistence tends to increase in arithmetical progression, the author says he does not see how this can be established.

In studying the assessment of taxes for Prussia, and also for other countries that have adopted the same method of collection, M. Pareto arrived at the following law: If N represents the number of individuals who have an income over x, then

$$N = Ax^{-\alpha}e^{-\beta x},$$

or less exactly (β being very small),

$$N = Ax^{-\alpha},$$

A, α, β being independent of x and of N.

It is stated that it is probable that this formula is applicable to all countries, and some arguments are given in favor of the law.

To the reviewer one of the most interesting sections of the book consists in a mathematical argument from which it results that under free competition prices are determined to the greatest satisfaction of all concerned in the transactions. This argument is based on the assumption that an individual retires from a market satisfied when a certain mathematical relation

$$\phi(q_1, q_2, \cdots, q_s) = a$$

exists between the quantities of merchandise $q_1, q_2, \cdots, q_s$ which he is engaged in exchanging. The value of this assumption might be questioned. The function $\phi(q_1, q_2, \cdots, q_s)$ is called by M. Pareto the ophélimité of the individual considered. Under a monopoly, the prices are again mathematically determined but not to the greatest satisfaction of all engaged in the transaction, but to the advantage of the proprietors of the monopoly. If there is a maximum of "ophelimité," the cost of production is equal to services rendered in production.

Many other applications are given in the book. Among these should be mentioned Cournot's theory of exchange, the rôle of the theory of games of chance in statistics, which includes the questions of annuities and insurance.

Taken as a whole, the book is useful for the clearness of presentation as well as for the numerous applications to economic theory. While the reviewer would expect a treatise on statistics to contain more recognition of the recent work of Karl Pearson and those associated with him, the present book contains much valuable material for the student of mathematical statistics.

H. L. RIETZ.

Σώζειν τὰ Φαινόμενα. Essai sur la Notion de Théorie physique de Platon à Galilée. Par P. DUHEM. Paris, A. Hermann et Fils, 1908. 144 pp.

OFTEN, when fatigued with the perplexities of modern physics or the intricacies of modern mathematics, it is a pleasant change to take a dilettante interest in the science of the ancients, to draw an optimistic courage from the progress twenty centuries have made or a pessimistic cheer from the little that so long a time has won. Then a volume of Pliny or parts of Plutarch's works suggest themselves — in a translation, alas! despite or to spite eight years of Latin and six of Greek. There we can find a dissertation on flesh eating which reads like some of all too recent date or a disquisition on the moon and her inhabitants that seems quite modern Martian. The philosophers who live much by and with and for the Greeks have collected, collated, and translated the words of philosophic wisdom of these ancients. If such a collection should be made for science with some appropriate comments relative to our present point of view, a highly entertaining book could be printed. Perhaps Duhem will sometime get to this; his present work with its Greek title and French subtitle is merely an essay on the conception of physical theory from Plato to Galileo — and by physical theory is apparently meant only such as regards astronomy, the best developed of the Greek physical sciences.