Die Theorie der Besselschen Funktionen. Von PAUL SCHAFHEITLIN. Leipzig, Teubner, 1908. v + 129 pp.

B. G. TEUBNER is publishing a series of mathematical and physical monographs for engineers. Each volume treats some topic in mathematics or in physics and in a hundred pages gives the busy engineer a general idea of the subject and a short account of its history, proves some theorems and gives others unproved, tells him something of the applications and gives him references to more complete works.

The volume under review is number four of the series. It is designed to give a reader who knows only the elements of the calculus a working knowledge of the theory of Bessel's functions. The introduction gives a short history of the functions, a list of important articles and treatises, and mentions the applications to hydrodynamics, wireless telegraphy, vibrations of a membrane, etc. Then comes a collection of some thirty unproved theorems and formulas, chiefly in hypergeometric series and gamma functions, to which the author refers in the course of the book. The subject matter proper begins with a solution of Bessel's differential equation in series by undetermined coefficients, and a definition of Bessel's function of the first kind. Since no knowledge of differential equations is assumed on the part of the reader, a short discussion of linear homogeneous differential equations and their solutions is given. Then follow with proofs the usual formulas giving the relations between Bessel's functions of different orders and their derivatives and the different expressions for the Bessel's functions of the first kind. In particular the case in which the parameter is an integer and the case in which it is the half of an odd number are discussed. The remaining chapters take up Bessel's functions of the second kind, the representation of arbitrary functions by Bessel's functions, the addition and multiplication theorems and the variation, graphical representation, and roots of the functions for various orders. Throughout the book important theorems and formulas are numbered in heavy type and these collected together form an appendix to the volume. The book is a mine of information about Bessel's functions, but the engineer will wish for a fuller discussion of the applications.

A. R. CRATHORNE.