THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

The one hundred and forty-seventh regular meeting of the Society was held in New York City on Saturday, February 26, 1910, extending through the usual morning and afternoon sessions. The following twenty-eight members were present:

Professor G. D. Birkhoff, Professor C. L. Bouton, Professor E. W. Brown, Professor F. N. Cole, Dr. Elizabeth B. Cowley, Professor L. P. Eisenhart, Professor Peter Field, Professor T. S. Fiske, Dr. C. C. Grove, Professor J. I. Hutchinson, Dr. L. C. Karpinski, Professor Edward Kasner, Mr. W. C. Krathwohl, Professor J. H. Maclagan-Wedderburn, Mr. A. R. Maxson, Mr. H. H. Mitchell, Professor G. D. Olds, Professor W. F. Osgood, Mr. H. W. Reddick, Mr. L. P. Siceloff, Professor D. E. Smith, Professor P. F. Smith, Dr. W. M. Strong, Dr. Elijah Swift, Professor E. B. Van Vleck, Professor Oswald Veblen, Mr. H. E. Webb, Professor H. S. White.

Ex-President W. F. Osgood occupied the chair at the morning session, Vice-President J. I. Hutchinson at the afternoon session. The Council announced the election of the following persons to membership in the Society: Mr. E. S. Allen, Berkshire School, Sheffield, Mass.; Mr. B. A. Bernstein, University of California; Mr. G. W. Evans, Charlestown High School, Boston, Mass.; Mr. C. E. Flanagan, Wheeling, W. Va.; Mr. C. E. Githens, Wheeling, W. Va.; Mr. J. S. Mikesh, University of Minnesota; Professor G. P. Paine, University of Minnesota; Mr. W. L. Putnam, Boston, Mass.; Mr. V. M. Spunar, Pittsburg, Pa. Nine applications for membership in the Society were received.

Committees were appointed by the Council to arrange for the coming summer meeting and to report on the subject of the publication of the Princeton Colloquium Lectures.

The following papers were read at this meeting:

(1) Professor G. D. BIRKHOFF: "A simplified treatment of the regular singular point."

(2) Professor G. D. BIRKHOFF: "Some oscillation and comparison theorems."

(3) Professor P. F. SMITH: "On osculating bands of surface-element loci."

(4) Professor EDUARD STUDY: "Die natürlichen Gleichungen der analytischen Curven im euklidischen Raume."

(5) Professor G. A. MILLER: "Addition to Sylow's theorem."

(6) Professor PETER FIELD: "On the circuits of a plane curve."

(7) Professor C. L. BOUTON: "Examples of transcendental one-to-one transformations."

(8) Professor JACOB WESTLUND: "On the fundamental number of the algebraic number field $k(\sqrt[p]{m})$."

(9) Professor L. P. EISENHART: "Surfaces with isothermal representation of their lines of curvature, and their transformations (second paper)."

(10) Professor EDWARD KASNER: "Isothermal nets."

(11) Dr. ARTHUR RANUM: "On the principle of duality in spherical geometry."

(12) Dr. O. E. GLENN: "On multiple factors of ternary and quaternary forms: applications to resolution of rational fractions."

In the absence of the authors the papers of Professor Smith, Professor Study, Professor Miller, Professor Westlund, Dr. Ranum, and Dr. Glenn were read by title. Professor Study's paper will appear in the July number of *Transactions*. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. This note by Professor Birkhoff is to appear in an early number of the *Transactions*. The theorems of Fuchs concerning the solutions of ordinary linear differential equations in the vicinity of a regular singular point are proved by a very short analysis which does not involve the substitution of an infinite series in the equation.

2. In a second note Professor Birkhoff derived oscillation and comparison theorems for ordinary linear equations of the third order analogous to the well-known one for second order equations. This paper will be offered to the *Annals of Mathematics*.

3. Professor Smith's paper is a continuation of that presented by the author at the summer meeting of 1909. The new matter consists chiefly in the discussion of cubic bands in space, that is, unions of ∞^1 surface elements whose points lie on a skew cubic and whose planes envelope a quadric cone. The vertex of the cone must lie on the cubic. If this cubic is

required to pass through three fixed points, then a cubic band is uniquely determined by two united surface elements. The point of contact with the theorems established in the former paper lies in this last statement. The cubic bands which osculate the characteristic bands of a partial differential equation of the first order osculate also the characteristic bands of a second differential equation. Other theorems analogous to those proved in the first paper hold for cubic bands also. Incidentally, a simple contact transformation appears, under which parabolic bands become cubic bands, vertical parabolas become four-nodal cubic surfaces, and straight lines become quadric cones.

5. According to Sylow's theorem the Sylow subgroups of order p^m are transformed according to a transitive substitution group. Professor Miller's paper deals with the question whether this transitive group is primitive or imprimitive. His main theorem may be stated as follows: If a group G contains more than one Sylow subgroup of order p^m and if one such subgroup has no more than p^α substitutions in common with any of the others, then G cannot transform these subgroups according to an imprimitive group unless their number is of the form $(1 + kp^{m-\alpha})(1 + lp^{m-\alpha})$, where both k and l are greater than zero. As a direct corollary we have that a group cannot transform its Sylow subgroups of order p^m according to an imprimitive group unless the number of these subgroups is of the form $(1 + kp)(1 + lp)$ where both k and l are integers greater than 0. For instance, if the order of G is divisible by 7, its Sylow subgroups of order 7^m cannot be transformed according to an imprimitive group unless their number is at least 64, and if it exceeds this number it must be at least 120, etc. This theorem is frequently very useful to determine whether a given substitution group is primitive.

6. It is known that there exist curves for every order n, $p = 0$ or 1, composed of a single circuit of index $n - 2$. In case $p = 1$ the curve may also have a simple oval (C. A. Scott, *Transactions*, volume 3). Professor Field's paper is devoted to a proof that there exist for every order n, $p = r$ $(r = 1, 2, 3, \cdots, n - 2)$ curves composed of r circuits, the sum of whose indices is $n - 2$. There may also be in addition a simple oval.

7. In this paper Professor Bouton gives a method for constructing a general class of transcendental one-to-one transfor-

mations in n variables, $n>1$. To his knowledge no examples of such transformations have heretofore been given. A simple example is

$$u = x + e^{y+\sin x}, \quad v = y + \sin x,$$

with the inverse

$$x = u - e^{v}, \quad y = v - \sin(u - e^{v}).$$

The method also gives classes of transcendental one-to-one contact transformations in n variables.

8. In Professor Westlund's paper the algebraic number field $k(\sqrt[p]{m})$ generated by the real pth root of a number m, where p is any odd prime, is discussed. An integral basis and the fundamental number of k are determined.

9. In a former paper with the same title (*Transactions*, volume 9) Professor Eisenhart established a transformation of surfaces S with isothermal spherical representation of their lines of curvature into surfaces of the same kind, such that upon a surface S and a transform S' the lines of curvature correspond, and S and S' constitute the envelope of a two-parameter family of spheres. This transformation was established by means of the Thybaut transformation of minimal surfaces and the transformation of partial differential equations of the second order due to Moutard. In the present paper transformations of this kind are established in a more simple and fundamental manner by determining the W-congruences (that is, congruences upon whose focal surfaces the asymptotic lines correspond) which are of such a character that the Lie line-sphere transformation converts the focal surfaces of the congruence into surfaces, S and S', with isothermal spherical representation of their lines of curvature. At the same time, S and S' are then necessarily the envelope of a two-parameter family of spheres. This method seems to give a more satisfactory reason for the existence of such transformations. The analysis is very much simpler in this case, and one obtains almost immediately a "theorem of permutability" for transformations of this kind.

10. Professor Kasner obtains the conditions which must be satisfied by two analytic curves intersecting orthogonally in order that they may be regarded as members of an isothermal net; that is, in order that they may be conformally transformable into orthogonal straight lines. The conditions are expressed

as an infinite set of relations between the coefficients of the power series representations of the given curves. All the coefficients of one series and half the coefficients of the other series may be taken at random, the other half then being determined by the general relations. Geometric interpretations are obtained by introducing the successive derivatives of the radii of curvature with respect to arc.

11. In the geometry of the surface of a sphere the principle of duality applies to metrical as well as projective properties. The consequences of this elementary fact do not seem to have been utilized much in the past. In this paper Dr. Ranum shows how the duality between distance and angle, between arc and area, between rolling and sliding, immediately leads to some interesting new theorems. For example, any continuous spherical movement can be regarded as due to the sliding (without rolling) of one spherical curve on another; the locus of a point lying on a variable tangent to a curve at a distance of $\frac{1}{2}\pi$ from the point of tangency is the envelope of the polar of the center of curvature at a variable point of the same curve; the area swept out by a moving tangent to one of two polar curves is equal to twice the length of the corresponding arc of the other.

12. In a paper published in the *American Journal of Mathematics*, volume 32, number 1, Dr. Glenn developed the theory of the decomposition of ternary and quaternary forms into simple quadratic factors. In the present paper this theory is extended to the cases where multiple factors occur. The multiple linear factors and multiple quadratic factors of a form are completely determined, save as to the solution of certain ordinary algebraical equations of higher degree.

In a footnote in the before-mentioned paper a new partial fraction theorem was announced, the unique feature of which was that the numerators of those partial fractions of b_x^{m-1}/a_x^m which have simple quadratic denominators were determined by ordinary differentiation and application of the Aronhold operator $(b \cdot \partial/\partial a)$. This theory is brought to a degree of completion in the present paper. Differential operators are developed by which are determined the numerators of the fractions corresponding to multiple quadratic factors of a_x^m. This theory is also extended to include the case where b_x^{m-1} and a_x^m are ternary forms.

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Secretary.