SHORTER NOTICE.

Allgemeine Formen- und Invariantentheorie; Band I. Binäre Formen. By W. Fr. MEYER. Leipzig, Göschen (Sammlung Schubert, volume XXXIII), 1909. viii + 376 pp.

IN accordance with the general plan of the Schubert series, the treatise on invariants commences with concrete examples to lead up to general theorems, presupposes no previous knowledge of the subject, yet presents a systematic discussion which includes all the essentials of this important discipline.

The entire theory of the quadratic equation, as developed in elementary algebra, is reproduced in all detail, and the same ideas are applied to systems of equations. Anharmonic forms, involutions, Jacobians, are all explained and illustrated.

Now come linear substitutions, first translations, then inflations, and finally reciprocations. The general substitution is shown to be made up of these three, and those functions which are unchanged by the three elementary operations are therefore invariant under the general substitution. Conversely, the linear substitutions generate groups which may be classified as one, two, or three parameter groups, according to Lie.

Properties of the self corresponding elements of a general substitution and of the double elements of an involution are treated in a manner that makes this chapter a valuable appendix to a course in projective geometry.

The preceding elementary and very concrete discussion occupies 118 pages; it is followed by a chapter of 40 pages on bilinear and quadratic forms, with an introduction of the concept of the differential operator. Here again every transformation is built up from the elementary ones, the effect of each operation upon functions of the coefficients being minutely examined. As an appendix some twenty pages are now added on symbolic representation, no use of which is made in any part of the book except in the appendix at the end; in the latter the fundamental theorem is proved that every invariant can be expressed symbolically in terms of the elementary determinant forms. While this theorem is perhaps desirable for the sake of completeness, it is presented from such a different point of view that the discussion is out of harmony with the rest of the book. The treatment is so concise that the proof of the theorem will hardly be convincing to the reader, let alone any possibility of using the new method in his own work.

The second part is concerned with the differential equations connected with invariants of binary forms. Given a function $J(a_1, a_2, \cdots)$ which is invariant under the transformation $A_1(h) \equiv x_1 = \xi_1 + h_1\xi_2,\ x_2 = \xi_2$ by which

$$f(x) \equiv \sum_{i=0}^{n} n_i a_i x_1^{n-i} x_2^i$$

goes into

$$\phi(\xi) \equiv \sum_{i=1}^{n} n_i \alpha_i \xi^{n-i} \xi_2^i,$$

n_i being the binomial coefficient $n!/(n-i)!\,i!$; if $J(\alpha) = J(a)$ under $A_1(h)$ and $A_2(h)$, it is called an invariant of translation. Since α_i is a function of h as well as the a, we have

$$J(\alpha) = J(a) + h \left.\frac{dJ(\alpha)}{dh}\right]_{h=0} + \cdots,$$

from which $J'(\alpha)_{(n=0)} = 0$.

From this equation the first invariant operator

$$\nabla_1 = \sum_{i=1}^{n} i a_{i-1} \frac{\partial}{\partial a_i}$$

follows, and similarly the second,

$$\nabla_2 = \sum_{i=0}^{n} (n-i) a_{i+1} \frac{\partial}{\partial a_i}.$$

There follows a good discussion of the laws of combination of these operators and their application to a series of simultaneous forms.

The concepts of order and weight, commutators, and infinitesimal transformation are now taken up and everything reduced to the fundamental operators. There are numerous examples given for the student to work out; the algebraic work in each illustrates the idea nicely, but many could have their value increased by showing their geometric meaning, as was done in the preceding chapter.

The second chapter in the second part is concerned with relative invariants. The theorem that all the coefficients of a binary form lacking the second term are relative invariants is given an elegant proof. It is shown how to construct forms of a given order and weight and that every invariant form for f_n is the source of a covariant for f_{n+r}. All these results are generalized to apply to a system of simultaneous forms.

The concept of an invariant is extended to apply to certain transcendental forms, including the logarithm and the elliptic integrals. By means of the former it is shown that every symmetric function of the roots can be rationally expressed in terms of the sum of the powers of the roots and a number of related theorems are derived (Waring's formulas). It is now easy to derive the expressions for the discriminant of an equation, the resultant of two such equations, and the expressions for the elementary relative invariants in terms of the roots. A second volume is in preparation which is to extend the preceding theory to ternary and quaternary forms.

To students of analytic geometry and of algebraic functions Professor Meyer's treatise will be of real assistance.

VIRGIL SNYDER.

NOTES.

THE April number (volume 11, number 2) of the *Transactions of the American Mathematical Society* contains the following papers: "The theorem of Thomson and Tait and natural families of trajectories," by EDWARD KASNER; "The introduction of ideal elements and a new definition of projective n-space," by F. W. OWENS; "The groups of classes of congruent quadratic integers with respect to a composite ideal modulus," by ARTHUR RANUM; "A simplified treatment of the regular singular point," by G. D. BIRKHOFF; "The strain of a gravitating, compressible elastic sphere," by L. M. HOSKINS.

AT the meeting of the London mathematical society held on March 10 the following papers were read: By W. F. SHEPPARD, "Forms of the remainder in the Euler-Maclaurin sum formula"; by J. W. NICHOLSON, "The scattering of light by a large conducting sphere"; by Miss H. P. HUDSON, "The 3–3 birational space transformation."

THE following papers have been read at recent meetings of the Edinburgh mathematical society. January 19: by R. SANGANA,