A TREATISE ON PHYSICS.

Traité de Physique. Par O. D. CHWOLSON. *Ouvrage traduit sur les éditions russe et allemande* par E. DAVAUX. *Edition revue et considérablement augmentée* par l'Auteur, *suivie de Notes sur la Physique théorique* par E. et F. COSSERAT. Tome 1: *Mécanique. Etats de la matière. Acoustique.* vii + 1092 pp. 1908. Tome 2: *L'Energie rayonnante.* 1188 pp. 1909. Paris, A. Hermann.

DESPITE the fact, and perhaps for the very reason that teachers of the first general collegiate course in physics, which usually falls in the sophomore year, are widely and chronically of the opinion that there exists no text suited to their needs, there are upon the market a great variety of elementary general treatises on physics, varying from a moderate volume of three or four hundred pages to lengthy works of three or four times the size, like that old standby Ganot. When we come to more advanced compendia, suitable for the maturer student of physics, we have not many to choose from, and none, so far as we recall, in the English language. Indeed it might well be questioned whether, with the present extended and rapidly growing domain of physics, it would be possible for a single author properly to cover the whole ground, whether it might not be necessary to fall back upon collaborations as in Winkelmann's Handbuch der Physik. We may remember that over ten years ago A. Gray printed a first volume of a Treatise on Physics,* which in its masterly manner gave promise of being just what was needed in this line; but apparently the work has been abandoned, much to our disappointment. However, Chwolson has succeeded in carrying a similar task, monumental as it is, to an eminently satisfactory conclusion.

So long as Chwolson's work remained in the original Russian it could be of little use outside of Russia. But as surely as a first class work appears the Germans, often in the guise of the house of B. G. Teubner, are apt to find it out and to publish a translation; this time, however, it was the firm of Vieweg und Sohn who rendered the German and international public the service of printing Chwolson's treatise in German. That

* Reviewed in this BULLETIN, volume 8, p. 403.

Hermann should continue the international conquest of the work is testimony not only to its high value, but to his own courage; for the French seem none too prone to take kindly to foreign texts, and the undertaking of a four volume treatise on physics, to run into thousands of pages and to sell at about 150 francs, is no light tax on one's faith and one's resources. It is with great pleasure that we observe that the demand for the work has been so great as already to necessitate a second edition, at least of the earlier fascicules.* As the French edition is the latest, it is also the best; for the author has been able to revise and elaborate. The translator has added some notes, and E. and F. Cosserat have furnished additional contributions.

The work commences with a general introduction which shows that Chwolson is somewhat of a philosopher. It is probably true that everybody who has large views on a wide range of phenomena must have some sort of philosophy which is interesting to him to expound and interesting to others to read, but which in the last analysis is rather a personal matter with which almost everybody else may differ. For instance the author states that the more limited the number of hypotheses, the more advanced the state of science. This is a beautiful ideal that appears to have little realisation in practice. From the opposite point of view one might just as well hold that the more advanced the state of science, the greater the diversity of hypotheses, and the better known the limitations or field of application of each. The hypothesis of imponderable caloric has never disappeared, and probably never will, from the theory of the flow of heat. To be compelled to treat heat as molecular motion in this case would be entirely too inconvenient. Some physicists now desire to abolish the hypothesis of an ether. They cannot do it; the best they can do is to formulate a new hypothesis to organize phenomena which are not satisfactorily organized under the ether-hypothesis. But as our philosophy cannot be as valuable to our readers as the author's should be to his, we should be lacking in humor if we persisted.

An elaborate discussion of physical quantities or magnitudes of different sorts and of units (not the theory of dimen-

* What was tome 1, fasc. 1, of 407 pages in 1906 has now become in 1911 tome 1, vol. 1, considerably larger. (I am reviewing from the earlier edition, which has only recently come to me for that purpose. E. B. W.)

sions) is a valuable contribution to the introduction. This is a subject which gives many students considerable trouble, but should not worry readers of this work. The notion of density, for example, is used in very different ways in physics, and needs the careful treatment the author gives. We must observe, however, that for all his care he defines ordinary density as it is ordinarily defined, namely, as a limit, whereas in reality on the molecular hypothesis no such limit can exist. This is but another instance of the diversity of hypothesis; no one disputes the molecular view of matter, yet few take the precaution to observe that the contradictory hypothesis of a continuum is constantly being used. We have no objection to this plurality of hypotheses, but we would not slur it over.

The exposition of the relative fields of experimental, theoretical, and mathematical physics is lucid. It is doubtful, however, whether the mathematical method does not encroach more upon and is not more necessary to theoretical physics than seems to be implied on page 11; and indeed the work as a whole seems to give more space to mathematical physics* than one might imagine from the introductory statements. It should be pointed out that mathematics is used in physics not merely to deduce consequences from well established facts, but to formulate new hypotheses and enquire after their consequences. In this latter sense much mathematical physics is as truly experimental as is considerable laboratory work.

The introduction closes with some mathematical work on differentiation, curvature, and vector addition. This is necessarily very meager and will be sufficient only for the more elementary parts of the text, some portions of which require very considerable mathematical training and ability. As references to original sources are very numerous throughout the remainder of the work, and form a very valuable part of it, the author inserts here a list of the most important periodicals cited, with their appropriate abbreviations.

Some elementary mechanics comes next—a little kinematics, rectilinear and curvilinear, the analysis of inertia, mass, and force according to Newton's laws, momentum, impulse, resultants, couples, center of inertia, and moment of inertia. The principle of the conservation of mass is not mentioned without adding a list of references to various recent experi-

* Due mainly to additions by translator and editors, and unfortunately of the French non-physical type.

ments attempting to detect changes (not, of course, the very recent experiments in connection with moving charges and the principle of relativity). The author states that there is a very general notion that Newton's second law refers to rate of change of momentum, and that he cannot share that opinion, but regards the law as dealing with acceleration. This same point of view is taken by Jouguet in his searching historical study of mechanics.* The English attitude, however, has seemed to incline more to the rate of change of momentum; and it may be interesting to note that it was by going back to the definition of force as rate of change of momentum that G. N. Lewis was able in studying relativity to assign a definite mass to a particle, instead of the varying masses from transverse to longitudinal that previous writers had used.

The author refers for the determination of centers of inertia to earlier and more elementary studies of the reader; in finding moments of inertia he uses triple integration without apology, except to say that readers unacquainted with the method may take the results for granted. When he goes on to treat work and energy, he uses sums, limits of sums, and simple integrals, even line integrals, without hesitation. We cannot help wondering how necessary it is to develop the small amount of differential calculus above mentioned if one is to go right on with integral signs. It seems as though it might have been better either to give a considerable account of mathematical analysis or to have taken the whole of elementary calculus for granted. In studying energy, the various forms (not merely the kinetic and potential energies of mechanics) are taken up and the three principles, namely, that the energy is a determinate function of the state, that energy is conserved, and that it may possess a direction of transformation, are elaborated.

The treatment of harmonic motion is detailed, including the composition of vibrations, damped oscillations, propagation of a disturbance by longitudinal or transverse waves, wave surface and ray, Huyghens's principle, reflexion, refraction, diffraction, and Doppler's principle. As the material covers over 60 pages, it is readily seen to be sufficiently emphasized. The author next proceeds to study universal gravitation and the newtonian potential with the minimum amount of mathematics, parabolic and pendulum motion in the field of gravity,

* E. Jouguet, Lectures de Mécanique, reviewed in this BULLETIN, vol. 18, p. 32, without mentioning this particular point.

and the theory of dimensions as applied to mechanics. There seems to be no need of any other comments than these: The treatment of the ordinary mechanics of particles and rigid bodies seems very meager and the use of mathematics extremely limited for a work of this size, which itself subsequently gives considerable development to the mechanics of continuous media. In this particular, the work appears unbalanced and inferior to Gray's Treatise on physics. We should recommend the addition of some theoretical mechanics with Gray as a model, although it would not be necessary to carry the matter so far as he does. The bibliography on mechanics is extensive and well chosen; but in regard to this list, and most others in the course of the work, we have been wholly unable to see why the author or editor did not make the arrangement according to date or alphabetical according to author or title instead of tossing the references together apparently pell-mell or following the order in which they were cited in the text.

We have said that as far as mechanics is concerned Chwolson's work is unbalanced; but we will hasten to add that this may not be due so much to his own doings as to his polite complacency in accepting the additions of others. For right here (pages 236–273) we come upon a long note by the Cosserats upon the dynamics of a point and of a rigid body, which has absolutely no place in this work, and should have had much less space even if the author had developed his mechanics to include Lagrange's equations and Hamilton's principle. We postpone further comment.

The following 125 pages are given to instruments and methods of measurement. To one not at all an experimental physicist this makes clear, pleasant, and instructive reading. There is good general advice as to methods of measurement, the elements of least squares, formulas of approximation, numerous diagrams of instruments designed to make mechanical (as distinguished from thermal or electric) measures, and ample explanation. Even the experiments to determine the mean density of the earth are discussed. (Why Burgess is not mentioned in this connection we do not understand.) There is a very appropriate note on the theory of integrating machines by the translator.

The second fascicule of Volume I deals with the gaseous state. If we are not mistaken, the author's choice of hydrogen ($H = 1$) as standard instead of oxygen ($O = 16$) is now somewhat old-fashioned. The experiments necessary for testing the equation $pv = RT$, and the results obtained, are naturally the first. In future editions of the book the work of P. W. Bridgman, who has succeeded in running pressures from 3,000 up to 12,000 atmospheres, will have to be cited with Natterer's, Cailletet's, and Amagat's on page 429. Clapeyron's formula and van der Waals's equation, with some of the modifications that have been suggested, are merely touched at this point. There follows a chapter on barometers, airpumps, and similar apparatus.

After some mention of the conduct of gases in the presence of other gases or liquids or solids (laws of Dalton, Henry, etc.), Chwolson takes up the kinetic theory and, while not going into very complicated mathematical analysis, is quite able to show the relations of the theory to the previously developed laws, and to discuss the mean velocity, mean free path, viscosity (Maxwell's law), and van der Waals's equation. Sutherland's formula for the viscosity is given without going into his theoretical deduction. Indeed it is characteristic of the author's good sense and balance to run through the development of such an important theory as the kinetic, and then add without any theoretical derivation some formula which has been found to fit the experimental data better than the one derived from that theory. The interesting chapter on the motion of gases (efflux, diffusion, resistance to projectiles, dissociation, etc.) is rendered somewhat unintelligible in our copy by the fact that pages 537–552 are omitted and 521–536 repeated through an error in binding, but enough can be seen to show that the interesting work of Hugoniot and recent formulas by Boussinesq are not overlooked.

A fascicule of over 300 pages is given to the properties of the liquid and solid state. For the most part the presentation is descriptive rather than mathematical or theoretical. Surface tension and capillarity receive considerable attention. The theory of electrolytic dissociation is mentioned, but the facts about solutions are here sketched without much attention to that theory, which is reserved for a later volume. In regard to osmotic pressure the author likewise refrains from adopting definitely any hypothetical explanation, contenting himself with emphasizing the difficulty of finding a satisfactory explanation by calling attention to several of those which have been suggested. Friction and viscosity are briefly discussed.

The treatment of the motion of liquids is partly descriptive, as might be found in a work on hydraulics, and partly mathematical, due perhaps to various additions by the translator. This makes the chapter read somewhat unevenly. Indeed it seems almost useless to include even the small amount of the theory of vortices here given unless the reader has had more thorough training in the composition of angular velocities than was suggested in the early work on mechanics. Moreover it is rather unfortunate, from the molecular point of view, to use such an expression as "the two molecules are always upon a vortex line" (page 711). It may be wondered whether this is Chwolson's own language.

The chapter on the colloidal state has evidently been written or at least edited by V. Henri, which is sufficient recommendation for it.

In regard to the treatment of the solid state there seems to be no need of comment. The same unevenness between the extremes of mere description on the one hand and considerable difficult mathematics of elasticity on the other is noticeable; indeed there are parts of the latter which would still seem of rather doubtful utility in this work even if the more extended treatment which we should recommend for mechanics were given.

In a great treatise on physics, like this, mechanics, hydromechanics, and elasticity should be introduced just so far as they develop and illustrate fundamental physical principles either in themselves or in their applications. The ideas of momentum, moment of momentum, and energy cannot be too highly emphasized, especially in view of Maxwell's theory of electromagnetics and modern theories on radiant energy. We should introduce the gyroscope (not here mentioned, so far as we observe*) because there is a lot of physics in it that can hardly be so well found elsewhere. We should discuss Lagrange's coordinates and Hamilton's principle because without these it is difficult to get the idea of a general dynamical system and of the most important single principle in mathematical (may we say theoretical?) physics. We should study hydrodynamics and elasticity for the physical principles involved, for the conceptions connected with dynamics in a medium, to emphasize the analogies and points of difference between liquid and solid media, the ether, and discrete systems.

* Except in connection with the gyrostatic theory of light!

We should bring to light and use general principles of reciprocity, and the like. But we should be rather wary of introducing mathematical developments which are devoid of general physical significance even though they lead to formulas of engineering or laboratory practice—unless we are frankly intent on including a good account of mathematical (as distinguished from theoretical) physics. It would not do to say that in the present edition of Chwolson's Physics there is too much mathematical work; but it is true that the physical side of a question seems almost immediately lost forever as soon as the analysis begins. This is a difficulty that is almost always in evidence in a French, as contrasted with an English, text; and if we rightly detect the uniformly physical spirit in which Chwolson himself writes, we cannot help feeling that the unevenness and mathematicity are not his.

These remarks apply also to a number of sections in the final fascicule (Acoustics) of the volume. What propriety is there in the sectional heading "Le problème général de l'acoustique. Conditions aux limites" in a work whose author in his introduction states that it is theoretical, not mathematical, physics that is important? It is hard to believe that boundary value problems really interest him. Still, although we do not feel that the translator and editors have made the best selection of additional mathematical material or have incorporated it in the smoothest fashion, we do not wish to give the impression that there may not be many readers of the work who will find these additions helpful and some who may find them the most helpful part of the treatise. There will be all sorts of readers with every diversity of requirement; and the editors probably had some particular class of readers in mind. Moreover, we have a feeling, and perhaps the translator had it, that purely mathematical physics is so widespread that some concession to it should be made. The average density of mathematics in the treatment of sound appears greater than in previous fascicules.

Although the usual topics are taken up, the author has space for a number of things not ordinarily mentioned, the propagation of finite displacements, explosive waves, where the velocities exceed the canonical. On the other hand, although diffraction of sound and the question of hearing are both treated, we do not find any mention of the way in which we perceive the direction from which the sound comes. It is interesting to note that in the section on longitudinal vibrations of strings the references are to the recent (1906) work of H. N. Davis. It is sometimes surprising how thoroughly the author has covered the literature up to the date of printing; it is by no means infrequent to see references dated within two years of the issue of the fascicule in which they are quoted. Apparently, however, the ten year old work of E. W. Scripture,* which would interest some students of sound, is not cited.

Volume I ends without an index; so does the next volume. If the complete work carries no index, or even if it carries only a poor one, a large amount of the usefulness of the treatise will be lost.

The second volume has the title Radiant Energy—a brave title. And the first fascicule displays a subheading Emission and absorption of radiant energy—a stalwart start. The complete adoption of the Maxwell theory, the firm belief in the reality of the ether and in its importance in the explanation of nature, the relentless grounding of the student in the use of a medium are characteristics of Chwolson's work which are more in evidence than in most other books and which rank his as a distinctly modern treatise.

And yet we of 1912 must admit that we cannot use the titles Radiant energy and Emission and absorption of radiant energy with the whole hearted confidence that we could wish, or that we could fairly assume five, or at any rate ten, years ago. We are in the unfortunate transitional stage of knowing too much and yet too little about radiant energy, its absorption, and its emission. We know too much to make the facts fit the Maxwell theory as we today understand it; we know too little or have too little genius as yet to found a new theory. There is a vast array of evidence that radiant energy appears usually, if not always, in discrete particles; there is a tendency to abandon the ether.

Chwolson should not be blamed for not taking toward the ether that attitude which has not yet ripened thoroughly and which was scarcely budding when the translation of his book began; rather should he be blamed if he had adopted it in place of the straight Maxwell theory, which even yet is none too clearly appreciated by the students to whom the author is talking. We cannot, however, withhold an adverse comment

* Yale Psychological Studies (1899), and his elaborate volume in the Yale Bicentennial Series (1901).

from the attempt to determine the density of the ether as about 10^{-17}; there are arguments (in England) just as valid for a density greater than the reciprocal of this; the whole subject is too far from settled to be given an apparently final conclusion in a text like this.

Much as we might desire to carry our review of this second volume along with the detail granted to the first, a detail which the work fully merits, we must refrain on account of the lengths to which it would lead. Moreover, the author has not been so often nor so vitally interrupted in his exposition by mathematical additions of the editors, and the text therefore appears so much more natural and homogeneous that comment is not so necessary. Finally, this volume on radiant energy and optics cannot readily and safely be judged apart from the subsequent presentation of electromagnetic theory in the fourth volume which we have not yet seen in completed form. We shall therefore run on rather rapidly, stopping merely to point out the general course of the development.

The start is strictly physical, with its description of the ether, of electric discharges and waves, of luminescence, and of radiation from hot bodies. Stefan's law, coefficients of absorption and emission, Kirchoff's law, Wien's law, Planck's formula, and the pressure of light are then treated. In dealing with the velocity of light the difference between wave velocity and group velocity is mentioned, as it should be. There is then a transition to topics in geometrical optics, reflexion, refraction, and the theory of mirrors and lenses. The facts about the index of refraction, including the case of metals, are presented with reference to various formulas which are stated without derivation. The subject is logically continued into dispersion and a lengthy discussion of spectrum analysis. There is a note by de Gramont on modern methods of spectrum observation.

After treating the transformation of radiant energy (fluorescence, phosphorescence, photography) and the measure of radiant energy (photometry, actinometry), the author goes over to optical instruments (microscope, telescope, and the like), the elements of physiological optics, and atmospheric optics. Not until then does he come to interference, and later to diffraction. These subjects, however, are covered in detail in some 120 pages. There is nothing of mathematical theory except a short note by the translator on retarded potentials and their application to problems in diffraction. In setting forth the phenomena connected with polarization the usual rather numerous trigonometric formulas are given without any complicated mathematical analysis. The translator, however, adds a note on Green's theory of reflexion and refraction based on the notion of an elastic medium and following Lord Kelvin's presentation in his Baltimore lectures. In this connection there is a short section on the gyrostatic theory of light.

The various sorts of elastic solid theory of light which have been developed from Green to Kelvin, have undoubtedly had very eminent adherents, have still a prominent place in optical theories of the more mathematical kind, and have certainly exercised a great influence over the development of optics. Their historical prominence is therefore such that they cannot be ignored. But it is difficult to repress the feeling that they have little place in the present work. And this for at least two reasons. First they were pretty thoroughly demolished by Gibbs's comparison of Kelvin's work with the electromagnetic theory. Second they do not accord with the strictly modern, maxwellian, point of view of the author. It may be that this discord will be remedied in the fourth volume. We should not have mentioned the matter were it not for the rather elaborate extension of old fashioned theories in the following treatment of crystalline optics. It may not be amiss further to state that from an elementary, though rather abstract, point of view it would have been better at this point to have founded the theory of light upon Gibbs's almost kinematic discussion of a wave motion in a fine grained medium. This too would have been somewhat nearer to the method of Boussinesq who, though generally ranked among the elastic solid theorists, is really very careful to indicate the ways in which the ether fails to act as a solid and to avail himself of general physical reasoning more or less independent of elastic theories.

The author's final three chapters on double refraction, interference of polarized light, and rotation of the plane of polarization, covering about 200 pages, are like most of the volume chiefly descriptive. Such subjects as accidental anisotropy and liquid crystals are not forgotten or slighted. The wonder is how Chwolson could so thoroughly and acceptably have collected and arranged and set forth such a vast variety of physical phenomena and theories as appear in the first half of his work. We must reserve the second half for notice at another time.

The volume should close here, but does not. For there is a note of 225 pages of the most mathematical character by the two Cosserats on the theory of deformable bodies. Even more than their note in the first volume is this wholly out of place, a worse than vermiform appendix. If the substance of these notes were unimportant, we could ignore them; but as they contain material which forms a distinct contribution to theoretical analytic mechanics, we shall try to give our readers an idea of their content at an early date.

From the numerous adverse comments which we have passed upon Chwolson's two volumes it might appear that we were not pleased with them. This is not so. We have found all the fault we could (a great deal of it with the editors' additions) in the hope that in future editions our criticisms might perhaps be productive of improvements. But without any changes the work is a masterpiece, and one that we should be glad to see not only in physical libraries but upon the shelves of our mathematical seminar rooms, to the end that our students of mathematics might readily avail themselves of its interesting and readable and broadening contents.

EDWIN B. WILSON.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
BOSTON, MASS.

ARITHMÉTIQUE GÉNÉRALE.

Je trouve dans le BULLETIN (Mars, 1912) un compte-rendu de mon livre Arithmétique générale.

Je crois nécessaire et de mon droit de rectifier une grave erreur du critique. Mon arithmétique n'est pas du tout un corollaire de la physique. J'ai dit au contraire que la mathématique pure doit être à mon avis une préface de la physique. Cela veut-il dire que la mathématique doive être une science expérimentale? Nullement. Or le critique donne à entendre que ce soit là ma façon de penser. Il dit que je démontre mes théorèmes fondamentaux en me servant de "remarques faites expérimentalement sur la nature des grandeurs" et le lecteur de ce compte-rendu doit s'imaginer