There is one disadvantage of this theory as compared to the ordinary analytic geometry. In the latter case the curve uniquely determines the (algebraic) function. But obviously we can find as many polynomials in x, y as we please, each passing through the same given points, finite in number. The real points on a modular curve $F \equiv 0$ do not therefore form an adequate picture of $F \equiv 0$. To this end and for the purpose of investigating intersections and all but the most trivial questions, we must introduce also the imaginary points of $F \equiv 0$, i. e., solutions of $F(x, y) \equiv 0 \pmod{m}$ in which x (and likewise y) is a root of any congruence modulo m with integral coefficients. The aggregate of the resulting infinitude of points gives an adequate representation of the function. If the author had recognized this point of view and had succeeded in materializing a suitable graphical representation of this infinitude of points, he would have made a substantial contribution to modular geometry. But in confining himself to real points, the author goes no further than earlier writers.* The author and his collaborators G. Tarry and Laisant are apparently not familiar with the history of Galois imaginaries, as there is no mention of Galois when such imaginaries (of the second order) are used and since a particular case of Galois' generalization of Fermat's theorem is attributed on page 148 to Tarry.

L. E. DICKSON.

La Logique déductive dans sa dernière Phase de Développement. Par ALESSANDRO PADOA. Paris, Gauthier-Villars, 1912. 106 pp.

THIS treatise is an adaptation of a course of lectures given by the author at Geneva, under the auspices of the university. The author had previously lectured on the subject in Brussels, Pavia, Rome, Padua, Cagliari, and presented memoirs before the congresses at Rome, Leghorn, Parma, Padua, and Bologna. The treatise contains an explanation, with abundant examples, of the symbols of logic as used in the Formulario Matematico, of Peano, some study of their properties, analysis of their relations, and their reduction to a minimum number. The author expresses his point of view very well in the following:

* Veblen and Bussey, "Finite projective geometries," *Trans. Amer. Math. Soc.*, vol. 7 (1906), p. 241. As the title shows, these authors were interested only in definite finite geometries and not in general modular geometry, so that the criticism of Arnoux's text does not apply to them.

"I do not hope to suggest to you the sympathetic and touching optimism of Leibniz, who, prophesying the triumphal success of these researches, affirmed: 'I dare say that this is the last effort of the human mind, and, when this project shall have been carried out, all that men will have to do will be to be happy, since they will have an instrument that will serve to exalt the intellect not less than the telescope serves to perfect their vision.' Although for some fifteen years I have given myself up to these studies, I have not a hope so hyperbolic; but I delight in recalling the candor of this master who, absorbed in scientific and philosophic investigations, forgot that the majority of men sought and continue to seek happiness in the feverish conquest of pleasure, money, and honors."

"Meanwhile we should avoid an excessive scepticism, because always and everywhere, there has been an élite—today less restricted than in the past—which was charmed by, and delights now in, all that raises one above the confused troubles of the passions, into the imperturbable immensity of knowledge, whose horizons become the more vast as the wings of thought become more powerful and rapid."

The symbols introduced are given below, with some examples. In the latter, N means integer, Np prime integer, $=$ (read *is the same thing as*) identity in the field of discussion, thus Rome = capital of Italy.

ϵ,	*is*, or *is a*, appurtenance of individual to class.	$7\epsilon Np$
$\subset$,	*contains*	
$\wedge$,	*impossibility, absurdity*	
$\vee$,	*true*	
$\supset$,	*is contained in*, inclusion of subclass by a class.	$15N \supset 3N$
$\smile$,	*smallest superclass*	$4N \smile 6N \supset 2N$
$\frown$,	*largest common class*	$4N \frown 6N \supset 12N$
ι,	*class of a single element*	$\iota 2 = Np \frown 2N$
$\rotatebox{180}{\iota}$,	*the only case*	$2 = \rotatebox{180}{\iota}(Np \frown 2N)$
$\ni$,	*such that*	$x \ni (x^2 + 26 = 10x) = \wedge$
',	*and*	
$.\supset.$,	*implies*	$x\epsilon a \,.\supset.\, x\epsilon b \;:=:\; a \supset b$
	two or more points may be used, thus $:\supset:$	

$. \frown .$, *simultaneous affirmation*

$$2 < x < 7 . \frown . 4 < x < 9 : = : 4 < x < 7$$

$. \smile .$, *alternative affirmation*

$$2 < x < 7 . \smile . 4 < x < 9 : = : 2 < x < 9$$

$\sim$, *negation* of what follows to a stoppoint

$$\sim (8 + 3 = 10), \quad 6 \sim = Np$$

$\exists$, *there are some* $\qquad \exists[N^2 \frown (N^2 + N^2)]$

Cls, class of (taken in extension).

Elm, class of only one element.

After explaining the significance of these signs, the author considers the properties of certain logical relations, such as equality, appurtenance, inclusion, implication, etc. The possible transformations of logical statements are considered, the figures of the syllogism developed, and finally he shows that all the other symbols can be defined by means of three, namely $=$, $\frown$, and $\ni$, in other words, in terms of the notions of *identity, largest common subclass,* and *such that.* Although such definition is possible, it is inconvenient to use nothing but these symbols, so that the others ought to be retained for convenience. The style of the treatise is clear and very simple, and as an introduction to the study of mathematical logic, can scarcely be excelled.

JAMES BYRNIE SHAW.

Elements of Plane and Spherical Trigonometry. By JOHN GALE HUN and CHARLES RANALD MACINNES. New York, The Macmillan Company, 1911. vii + 101 pp., with tables, pp. 102–205.

IN writing this book, the authors have undertaken "to present in as brief and clear a manner as possible the essentials of a short course in trigonometry." This aim they appear to have kept constantly before them. The language of the book is simple, concise, and interesting. The subject matter is brief enough to be covered by a class in somewhat less time than that usually required, and still comprehensive enough to take in all that is usually regarded as essential.

The authors have included one subject not often treated in a text book in trigonometry, namely, the drawing of graphs of equations in polar coordinates. For this, they give the