class m.   Hence $w_x = 0$ *cuts $C(n; m)$ in an nm-line,* by Bezout's theorem.*

If we project the curves (11), (12) upon the x_3 plane, we may obtain the equation of the projected *mn*-line by interchanging point and line coordinates in Clebsch's proof of Bezout's theorem (see Vorlesungen über Geometrie, page 282). Every full invariant of this *mn*-line gives by our translation principle an equation of condition among the coordinates w_i.

An alternative method of procedure is to use equations (1), (3), replacing f_1 by g_m in (1). Rational elimination processes give a form in each variable p_1, q_1, p_2, q_2 with coefficients rational in w_i. Of these forms that in p_2 is the transformed of the one in p_1, say of $F_1(p_1)$, by a homographic transformation, and that in q_2 is likewise the transformed of the one in q_1, viz. $\varphi_1(q_1)$. But φ_1 is not transformable into F_1. As an invariant of the *mn*-line of intersection we may then select a simultaneous invariant of the binary forms $F_1(p_1)$, $\varphi_1(q_1)$, and by translation this invariant goes into an equation of condition in the variables w_i, representing a contravariant surface.

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SOME MATHEMATICAL BOOKLET SERIES.

Matematica dilettevole e curiosa. Di ITALO GHERSI. Con 693 figure originali dell'Autore. Milano, Ulrico Hoepli, 1913. viii+730 pp.   Price L. 9.50.

Wo steckt der Fehler? Trugschlüsse und Schülerfehler. Gesammelt von Dr. W. LIETZMANN und V. TRIER. Mathematische Bibliothek, Nr. 10. Leipzig and Berlin, B. G. Teubner, 1913. 57 pp.   Price M. 0.80.

ENGLISH and French mathematical literature is entirely lacking in such admirable booklets dealing with elementary topics, as those which have wide circulation in Germany and Italy.† I refer to the Mathematische Bibliothek of the

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* Bezout, Theorié générale des Equations algébriques (1779).

† It may be suggested that the volumes on Elimination by Laurent and on Geometrography by Lemoine, of the excellent "Scientia" series (Gauthier-Villars, Paris) are elementary, but these are only two of a dozen volumes by Appell, Gibbs, Hadamard, Poincaré, etc., which certainly may not be classed in this way. And even these two brochures are more

Sammlung Göschen,* the Lietzmann-Witting Mathematische Bibliothek † and the mathematical volumes of the Biblioteca degli studenti,‡ and the Manuali Hoepli.

The Mathematische Bibliothek contains about 35 volumes ($4\frac{1}{4}$ x $6\frac{1}{4}$ inches; uniform price, $22\frac{1}{2}$ cents), each neatly bound in cloth and containing from 130 to 230 pages.   A. Sturm, H. Schubert, M. Simon, O. Th. Bürklen, K. Doehlemann and E. Beutel are among the authors and the volumes treat of History of mathematics, Plane geometry, Descriptive geometry (2 volumes), Determinants, Analytical geometry of the plane, Analytical geometry of space (notably fine figures), Projective geometry, Algebraic curves (2 volumes),§ Insurance mathematics, Vector analysis, Geodesy, Surveying, Astronomy, etc.

Of the Mathematische Bibliothek herausgegeben von W. Lietzmann und A. Witting a dozen volumes have already appeared.   They are bound in boards, contain 41 to 93 pages ($4\frac{3}{4}$ x $7\frac{1}{4}$ inches) each, and are of the same uniform price as the Göschen Sammlung before 1913.   In this series Wieleitner has written on the Idea of number in its logical and historical development; O. Meissner is author of Theory of probabilities with applications; M. Zacharias wrote the Introduction to projective geometry; Zühlke, Geometrical constructions in a limited plane; Beutel, Squaring the circle.

In the Biblioteca degli Studenti are nearly a score of volumes (4 x $6\frac{1}{4}$ inches; limp covers; single numbers of about 85 pages, 10 cents, double numbers of about 170 pages, 20 cents). They include, Manual of plane trigonometry, Manual of spherical trigonometry, Exercises of elementary geometry, Guide to the resolution of problems in algebra, Principles of perspective, Repertorium of mathematics and elementary physics, etc., and treat of very elementary topics.

Some 40 of the 1,200 odd volumes ($4\frac{1}{4}$ x 6 inches) in the Manuali Hoepli series are of mathematical content.   Perhaps the two best known works are the volumes (658+950

advanced in character than any of those in three, and than many of those of the fourth series, about to be considered.   The same may be remarked concerning the Cambridge Tracts in Mathematics and Mathematical Physics.

* G. J. Göschen'sche Verlagshandlung, Berlin und Leipzig.
† B. G. Teubner, Leipzig und Berlin, 1912–1913.
‡ Raffaello Giusti, editore, Livorno.
§ The second volume was reviewed by Professor White in this BULLETIN, vol. 19, pp. 417–419, May, 1913.

pages) of Pascal's Repertorio di matematiche superiori* since translated into German and enlarged,† and Pascal's Determinanti e applicazioni, 1897, which three years later was elaborated into a volume of the Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften. Then there are 4 volumes on Algebra, 4 on Arithmetic, 2 on Astronomy, 4 on the Calculus, including volumes on Calculus of variations and Finite differences,‡ and Critical exercises on the differential and integral calculus; 1 on Mathematical formulæ;§ Saccheri's Euclide emendato; 3 volumes on Functions (analytic, elliptic, polyhedral and modular‖); 13 on Geometry, 1 on the Mathematics of economics,¶ 1 on Groups, 4 on Mechanics, and 1 by G. Loria on Exact science in ancient Greece.** The volume of Ghersi under review is the second he has written for this mathematical series, the earlier one having dealt with Methods for resolving problems of elementary geometry.

In recent times English, French, and German writers have published popular works for recreation hours of those who are in any wise interested in mathematics. Ball's Mathematical Recreations and Essays, which has recently reached a fifth edition,†† is almost a classic in its special field. The older works of Lucas, "Récréations mathématiques"‡‡ and "L'Arithmétique amusante"§§ are frequently referred to,

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* Milano, 1898 and 1900. The first volume is reviewed by E. O. Lovett in this BULLETIN, vol. 5, pp. 357–362, April, 1899.

† Two volumes, Leipzig, 1900, 1901. New edition to be completed in 4 volumes; vols. 1_1, 2_1, 1910, reviewed in this BULLETIN by C. H. Sisam, vol. 19, pp. 372–374, April, 1913.

‡ Translated into German by A. Schepp. Leipzig, 1899. Reviewed by J. K. Whittemore in this BULLETIN, vol. 6, pp. 352–4, May, 1900.

§ This volume by Rossotti was reviewed by E. O. Lovett in this BULLETIN, vol. 5, pp. 261–2, Feb., 1899.

‖ This volume by Vivanti was reviewed by J. I. Hutchinson in this BULLETIN, vol. 14, pp. 144–5, Dec., 1907. The French edition by A. Cahen was reviewed by G. A. Miller in this BULLETIN, vol. 19, pp. 534–5, July, 1913.

¶ This volume by Virgilii was reviewed by J. M. Gaines in this BULLETIN, vol. 5, pp. 488–9, July, 1899.

** This work, which has just been published, 1914, contains about 1000 pages. The title page with "seconda edizione totalmente riveduta" is misleading, as the original work of over 900 pages of *quarto* format was a reprint of memoirs (in five books) in *Mem. Acc. Modena* (2), vol. 10, pp. 3–168; vol. 11, pp. 3–237; vol. 12_2, pp. 3–411; (1893–1902). Futhermore, "Libro II, Il periodo aureo della geometria Greca" appeared in still another form in *Mem. Reale Acc. d. Sc. di Torino* (2), vol. 40, pp. 369–445; (1890).

†† London, 1911.

‡‡ Paris, T. I, 2ᵉ éd., 1891; T. II, 2ᵉ éd., 1896; T. III, 1893; T. IV, 1894.

§§ Paris, 1895.

while the circulation of Ahren's Mathematische Unter-
haltungen und Spiele* and Schubert's Mathematische
Musestunden† is confined more to Germany. Each work
has its own peculiar ideals, but Ball is perhaps the most
comprehensive in range, while he and Ahrens alone introduce,
to an appreciable extent, references to the widely scattered
literature of the subject. E. Fourrey's "Curiosités géomé-
triques"‡ is also notably full in exact statement of authorities.

From works such as these, from books like Blythe's on
Models of cubic surfaces, Catalan's Théorèmes et problèmes
de géométrie élémentaire, Cremona's Elementi di geometria
proiettiva, Enriques' Questioni riguardanti la geometria elemen-
tare, de Longchamps' Essai sur la géométrie de la règle et de
l'équerre, Loria's Spezielle algebraische und transzendente
ebene Kurven, and from various periodicals, Ghersi has
compiled the present little work on Matematica dilettevole
e curiosa.

The first 74 pages are taken up with " Curious and bizarre
problems " such as: Euler's problem of the Königsberg
bridges, the Hampton Court maze and other unicursal
problems, map-coloring problem, and chess problems. Of
course little more than the statement of a problem is frequently
given.

In the next 100 pages various curious properties of numbers,
and problems of arithmetic and arithmetic geometry are set
forth. For example, we have properties of perfect and amic-
able numbers, of the triangle of Pascal, of Lucas's singular
products, as well as problems of Benedetti (Speculationes
diversæ, 1585) and of Leonardo Pisano (Liber Abaci, 1202).

Fermat's equation and other problems of the theory of
numbers are treated in the next 15 pages, then follows a
collection of miscellaneous algebraic problems which conclude
with graphical solutions of equations of the second, third and
fourth degrees and with a sketch of Demanet's and Meslin's
hydraulic,‡ and Lucas's electric solution of equations.

Magic squares, magic polygons, and magic polyhedra are
illustrated on pages 251–326.

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* Leipzig, 1901.
† Grosse Ausgabe, Leipzig, Bd. I, 3. Aufl., 1907; Bd. II, 2. Aufl., 1900.
‡ 2e éd., Paris, 1906.
§ Cf. " Two hydraulic methods to extract the nth root of any number "
and " Hydraulic solution of an algebraic equation of the nth degree,"
by A. Emch, *American Mathematical Monthly*, January and March, 1901,
vol. 8, pp. 10–12 and 58–9.

Then follow 350 pages treating of miscellaneous questions in geometry. On pages 329–367 we find definitions and derivation of properties, of notable transcendental, and cubic, quartic, and other algebraic curves. The next dozen pages contain instruments for tracing by continuous motion such curves as the conic sections, cissoid, and conchoid. Some 20 pages given over to discussion of the solution of problems in elementary geometry, by ruler and compass, and then (pages 407–422) cyclotomy is touched upon. Then come 100 pages occupied with the problems of trisection of an angle, squaring the circle, duplication of the cube. Dissection of figures, geometrical pavements, star-polyhedra, and hyperspace are some of the concluding topics under the head geometry.

In the final sections are paradoxes and other recreations in mechanics.

It will be remarked, as indeed the title implies, that the volume is not confined to so-called recreations, although these occupy the major part of the volume. It is written with light touch and anyone unacquainted with books on mathematical recreations may pass a few pleasant hours in turning over the pages and find some things not met with in other books of the kind. The reader who wishes to learn more of the underlying theory will then naturally turn for guidance to such a book as Ball's or to the article in the Encyklopädie* or to such works in fields other than those of recreations, as mentioned above.

In the Lietzmann-Trier Bändchen, which may be classed as a small addition to the literature of mathematical recreations, Lietzmann collected the 36 fallacies (Trugschlüsse) and Trier the 50 pupils' mistakes. Arithmetic, algebra, elementary geometry (synthetic and analytic), trigonometry are the only subjects illustrated. The errors in the reasoning are not indicated.

Among the fallacies are (1) numerous examples depending for their results upon division of each side of an equality by zero or neglect of consideration of double sign before a radical; (2) a series of geometrical paradoxes, several of which are already familiar through Ball's book.

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* Mathematische Spiele, von W. Ahrens, vol. I, 2, pp. 1080–1093, Leipzig, 1902.

Here is an example of a different kind, which appears to be new: " Consider

(1) $\log_e 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - \cdots;$

multiplying through by 2 we get

$2 \log_e 2 = 2 - 1 + 2/3 - 1/2 + 2/5 - 1/3 + 2/7 - \cdots.$

Collecting terms with common denominators and arranging according to increasing denominators, we get

(2) $2 \log_e 2 = 1 - 1/2 + 1/3 - 1/4 + 1/5 - \cdots.$

This is, however, the same as (1).   Therefore

$$\log_e 2 = 2 \log_e 2."$$

The examples of Schülerfehler are taken from the exercises of Danish pupils.   The vagaries of American youth suggest that an equally interesting collection could be made on this side of the water.   The error in No. 32 is not evident.   But here is No. 36: " Given two circles which cut one another in P and Q and touch the sides of an angle, on the same side of the vertex, at the points A, A_1 for one circle and B, B_1 for the other.   Prove (1) that PQ produced passes through the middle points of AB and A_1B_1; (2) that AA_1, BB_1 and PQ are parallel to one another."   Solution: " PQ cuts AB in C, A_1B_1 in C_1. Then by the power theorem, $CA^2 = CP \cdot CQ = CB^2$.   Therefore C is the middle point of AB.   In the same way C_1 is the middle point of A_1B_1. AA_1, PQ and BB_1 are parallel to one another because they cut off the equal segments on the lines AB and A_1B_1."

Finally, No. 47: " The sides of a triangle are a, b, c.   To express $\sin A$ in terms of the given quantities."   Solution: "Of course the following relations hold good:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

In a proportion it is allowable to interchange the means; hence

$$\frac{a}{\sin A} = \frac{b}{c} = \frac{\sin B}{\sin C}. \qquad \therefore \sin A = \frac{ac}{b}."$$

<div align="right">R. C. ARCHIBALD.</div>

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