

MATHEMATICAL MODELS.

Catalog mathematischer Modelle für den höheren mathematischen Unterricht. Veröffentlicht durch die Verlagshandlung von Martin Schilling in Leipzig, mit 106 Abbildungen. Siebente Auflage, Leipzig, 1911, xiv + 172 pp.

Verzeichnis von H. Wieners und P. Treutleins Sammlung mathematischer Modelle für Hochschulen, höhere Lehranstalten und technische Fachschulen. Zweite Ausgabe mit 6 Tafeln. Leipzig und Berlin, Verlag von B. G. Teubner, 1912, 64 pp.

Abhandlungen zur Sammlung mathematischer Modelle. In zwanglosen Heften herausgegeben von HERMANN WIENER. Leipzig, Verlag von B. G. Teubner. 1. Heft von H. Wiener, 1907, 90 pp.; 2. Heft von P. Treutlein, 1911, 20 pp.

Illustrierter Spezialkatalog mathematischer Modelle und Apparate. Entworfen von G. KOEPP und anderen bewährten Fachmännern. New York City, Eimer and Amend, 128 pp.

FOR fifteen years the bulk of the models used by advanced mathematical students all over the country has been procured from Schilling of Leipzig. This firm was developed from the Firma L. Brill of Darmstadt, the foundation of which reaches back some 35 years. Klein and A. von Brill, in those early years professors in the Technische Hochschule at Munich, had more than two score of models made for the Hochschule under their direction. Copies of these (for example: the tractrix of revolution, geodetic lines on an ellipsoid of rotation, Kummer's surface, forms of Dupin's cyclide, the spherical catenary, twisted cubics) in gypsum, wire, and brass form a portion of the great Schilling collection. In more recent times construction of new models has been carried on by the assistance of many other mathematicians. Among them are Professors Dyck, Finsterwalder, Kummer, Schoenflies, H. A. Schwarz, C. and H. Wiener.

As nearly ten years had passed since the sixth edition of the catalogue, the seventh edition† fills a long felt want. It

† Descriptions of models which have been manufactured since this edition was published, have appeared in *Jahresbericht d. Deutsch. Math.-Ver.*, 1913, vol. 22, pp. 75-76, 134-137.

describes some 400 models. Nothing more than an indication of the subjects illustrated can be given here: surfaces of the second order, algebraic surfaces of the third order, algebraic surfaces of the fourth and higher orders, line geometry, screw surfaces, space curves and developable surfaces, descriptive and projective geometry, analysis situs, algebra, function theory, mechanics and kinematics, mathematical physics, and structure of crystals,

Shortly after the third International Mathematical Congress at Heidelberg in 1904, Teubner offered to the public a selection of about 60 of the mathematical models for Hochschule instruction which had been exhibited at the Congress by the mathematical Institut of the Technische Hochschule of Darmstadt. The construction of the models in the selection was inspired by Professor H. Wiener.* In the new catalogue now before us we find that Professor Wiener has increased his collection by 50 models, while the late Professor Treutlein has contributed about 200 more.† All of the models are designed as aids to instruction in German secondary schools and Hochschulen. For students of higher mathematics the models of twisted curves and deformable quadric surfaces will probably be the only ones of especial interest.

The Abhandlungen are intended to be of value for those using the models. In Heft 1 are 9 Abhandlungen by Wiener: (1) Mathematical models and their use in instruction (pages 3-8); (2) On the projection of some plane figures (9-10); (3) The regular Platonic polyhedra, Regularity in a group (11-14); (4) Regular polygons and closed reflective systems (15-18); (5) The building up of the regular polyhedra (19-51); (6) How shall surfaces, especially those of the second order, be drawn? (52-54); (7) On surfaces of the second order (55-84); (8) Deformable thread models of ruled surfaces of the second order with fixed thread lengths (85-87); (9) Deformable metal-bar models for transforming a surface of the second order into confocal surfaces (88-91).

These Abhandlungen are similar to those which Schilling

* The catalogue (*Verzeichnis mathematischer Modelle*, 28 pp.) was published in 1905.

† An interesting account of these models written by Prof. H. Wiener, may be seen in *Jahresbericht d. Deutsch. Math.-Ver.*, Nov., 1913, vol. 22 pp. 297-304.

distributes* with his models and some of them are of considerable interest; on the one hand because of the developments of the theory of the surfaces, on the other through the application of the theory to construction of the models. Numerous bibliographical references are given. In illustration of these characteristics note, for example, (5) and (9).

In (5) the first five pages contain an historical review of the subject, then the theoretical considerations are treated under the headings: The notion of a polyhedron (e. g., Idea of a side, of "Vielkant," of "Vielflach," of "Vielzell"); First and second definitions of the regular polyhedron by the group; Transformation of an angle into a neighboring angle (by rotation); Range of the different suppositions; Third definition of the regular polyhedra; Construction of a regular polyhedron from its group.

In (9) we find that the construction of the model was made possible through theorems of Henrici and Greenhill. Among other studies in this connection, those of Mannheim, Darboux, and Schur are also considered.

The second Heft, written by Treutlein, contains *Abhandlungen* on the following subjects: "On the intuitive method of mathematical instruction" (pages 3-6); "On mathematical models and their use in teaching" (7-9); "Explanations in connection with the series, and the single models, of the Treutlein collection" (10-20).

In all of the above mentioned publications, Dyck's *Katalog*† is frequently referred to.

The Eimer and Amend collection is of use more particularly in connection with elementary work in planimetry, stereom-

* The "Erste Folge, *Abhandlungen zu den Serien I-XXIII, mit 71 Figuren auf 6 Tafeln und im Text*" have also been published in a single volume, in connected form. In the "Neue Folge" Hefte 1-9 have been already issued between 1899 and 1912. The authors are: Fr. Schilling, H. Wiener, W. Ludwig, H. Grassmann, W. Boy, E. Estanave, R. Hartenstein, and F. Pfeiffer.

† *Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente. Unter Mitwirkung zahlreicher Fachgenossen herausgegeben im Auftrage des Vorstandes der Deutschen Mathematiker-Vereinigung von geh. Hofrat Dr. Walther v. Dyck, Professor an der Technischen Hochschule zu München. Teubner, Leipzig, 1892, xvi+430 pp.*

Nachtrag, Leipzig, 1893, x+135 pp.
 These volumes contain papers by Klein, Voss, Brill, Hauck, v. Braunnühl, Boltzmann, Amsler, and Henrici, beside descriptions of the various models by their respective designers.

etry, trigonometry, and related branches. The models of star-polyhedra, Poinset polyhedra, and the so-called Archimedian semi-regular solids may be mentioned as desirable for more advanced mathematical considerations.

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DIFFERENTIAL GEOMETRY.

Leçons sur les Systèmes orthogonaux et les Coordonnées curvilignes. Par GASTON DARBOUX. Deuxième édition, complétée. Paris, Gauthier-Villars, 1910. 8vo. i+567 pp.

THE first 323 pages of the present volume constitute a reprint of the first edition, which was reviewed in the *BULLETIN* of January, 1899, by President E. O. Lovett. It was originally Darboux's intention to include in the complete work a number of other subjects, such as the theory of quadratic differential forms. This plan was eventually abandoned, and only such subjects are discussed in the completed volume which is now before us as are more or less directly connected with orthogonal triple systems of surfaces and families of Lamé. From the point of view of the artistic unity of the book, this change of plan is only to be commended. Let us hope however that the Fates may deal kindly with the great master of differential geometry; may the fear, which he expresses in his preface, of being prevented from completing his discussion of these other matters, prove to be unfounded; and may he be permitted to add many further contributions to the science to which he has devoted his life and which already owes him so much.

For mathematics owes a great debt to Darboux. Rarely do we find such a combination of the geometer and the analyst as is present in this master mind. He makes clear, not merely by precept but by example, that it is not enough to express a problem of geometry in analytic form and then solve the differential equations. He recognizes the true task of differential geometry as an exhibition of the complete parallelism between analysis and geometry. Not until this has been accomplished does he rest content with the solution of a problem, and many of his most notable contributions have resulted from this tendency to strive for a complete