G. M. Green.* There can be little doubt but that the methods of projective differential geometry will serve to throw a flood of light upon the theory of conjugate triple systems, which promises to become a most fascinating and fruitful field for further research.

E. J. WILCZYNSKI.

THE UNIVERSITY OF CHICAGO,
September 23, 1913.

DESCRIPTIVE GEOMETRY.

Elements of Descriptive Geometry, with applications to spherical and isometric projections, shades and shadows, and perspective. By A. E. CHURCH, late professor of mathematics in the United States military academy, and G. M. BARTLETT, instructor in descriptive geometry and mechanism in the University of Michigan. New York, American Book Company, 1911. 286 pages and 143 figures.

Practical Geometry and Graphics. By D. A. LOW, professor of engineering, East London College. London, Longmans, Green and Company, 1912. 448 pages and 823 figures.

Vorlesungen über darstellende Geometrie. By GUIDO HAUCK, late professor of descriptive geometry and graphical statics in the technical school at Berlin; edited by ALFRED HAUCK, director of the Realschule in Schönlanke. Leipzig, Teubner, 1912. Volume I. 639 pages and 641 figures.

Lehrbuch der darstellenden Geometrie für technische Hochschulen. By EMIL MÜLLER, professor of mathematics at the technical school of Vienna. Second volume, first instalment. Leipzig, Teubner, 1912. 129 pages and 140 figures.

MR. Bartlett's revision of Church's well-known Descriptive Geometry follows the original text very closely, the main departures being in mechanical make-up. The type is larger; the articles are renumbered; important theorems and new words are printed in bold-faced type; the figures are re-drawn and put in the text itself, a number of new ones being added, in particular, two hyperboloids tangent along a generator,

*G. M. Green, "Projective differential geometry of triple systems of surfaces." Dissertation Columbia University, 1913. Press of the New Era Printing Company, Lancaster, Pa.

development of a tea-pot spout, intersection of a torus and a cylinder as exemplified in a joint of a water pipe; a generous list of illustrative exercises for the student is distributed throughout the text. A few corrections in loose mathematical expressions have been made, but too many still remain, and a few of the new ones are also inaccurate. In all important features the text is still elementary and conservative. While it retains many of the antiquated features long discarded in the draughting room, still it does impart the essence of the subject in a lucid and concise way.

Professor Low's book commences with elementary constructions usually met with in books on plane geometry. The chapter on the circle is in sufficient detail for the actual beginner in geometry. Then follows one on geometrical conics, after which comes a short chapter on tracing-paper problems, In the introduction of each new idea the student is urged to perform all the steps of every construction. Chapter V considers approximate solutions of various graphical problems, their actual solution being impossible with ruler and compasses; they include rectification of a circular arc, squaring of the circle, obtaining the square root of π, and others. Roulettes and glissettes are treated by means of a combination of the methods of tracing paper and of approximate solutions. The short chapter on vertors contains little more than what one learns in a first course on mechanics; the principles are applied to graphical statics, including an extensive treatment of center of gravity and of moment of inertia.

Now follows a chapter of 20 pages on plane coordinate geometry, starting at the beginning and treating straight lines, conics, and spirals. A chapter on harmonic motion combines graphical with analytic methods in the discussion of finite series of sine curves.

Many topics are treated more thoroughly than even in the better American institutions; apart from the rule of thumb method of approximations, these 165 pages cover much the same kind of work as an American student would have had who had taken high school mathematics, a brief course in graphical algebra, and an elementary course in mechanics.

The subject proper begins with projection, first illustrated by a drawing in central perspective, then followed by the usual two-plane method of orthographic representation.

Treatment of points, lines, planes, prisms, pyramids, polyhedra, cones, and spheres is rapid, but clear and extensive. In the discussion of the uses of the general profile the drawings in h and v are accompanied in each case by a pictorial one which makes the problem much more real. The elementary principles are followed by numerous practical applications, including joinery, mitred mouldings, and a generous list of problems for the student.

A detailed discussion of the sphere and the quadric cylinder and cone follows, after which we return to the consideration of plane figures defined in various particular ways.

This second part of 114 pages completes the treatment of orthographic two-plane projections. It includes all the principles usually given in an ordinary course in descriptive geometry, treats a number of special cases, provides one or more figures for every case considered, and furnishes hundreds of exercises for the student to work out.

Now follows an excellently well written chapter on horizontal projection, including contours and various problems connected with them. In this chapter a knowledge of differentiation is presupposed. Now comes pictorial drawing, parallel perspective and central perspective, together with the relation between these methods and those of orthographic projection.

A combination of various preceding methods is now employed in the discussion of curved surfaces, tangent planes, developments, helices and screws, intersections of surfaces, together with a short introduction to shades and shadows. The book closes with a short chapter on miscellaneous applications to solid geometry. It is provided with an index and a number of numerical tables.

One who has mastered Professor Low's elaborate treatise will be well equipped with a knowledge of graphical processes and particularly with the relations between them, although no use is made of general axonometric methods; isometric projection is mentioned incidentally as a particular kind of pictorial drawing.

During a long period the lectures and exercises of Professor Guido Hauck at the technical school of Berlin have enjoyed a reputation for elegance, beauty, and pedagogical merit hardly to be found elsewhere. Under these circumstances it

is interesting to be able to see the book prepared on the results of this experience and to profit by the study of the methods of the master. The science of descriptive geometry is essentially an auxiliary one; its main purpose is to provide suitable tools for other purposes, namely, the proper representation of objects for the artisan and the engineer. In so far as it remains a science and does not become purely an empirical procedure it serves the further purpose of providing real mental discipline and spurs the students to originality in extending its scope and its usefulness.

The book commences with the usual presentation of the orthogonal two-plane projection, with the added feature of showing the reader how to distinguish which of two segments lies in front of the other—first of points, then of segments of lines, then of polygons, the latter incidentally determining a plane. True lengths of segments, size of triangles, shape and size of polygons are considered before the plane is mentioned. Then follow a number of problems concerning planes, defined by three non-collinear points, problems involving the angles which a line makes with the vertical and the horizontal planes, distance between parallel planes, etc. A large number of exercises are provided, but all of them are solved in the text, though in many cases not much more than an outline of the solution is given. In the constructions the ground line is always used, but the figures have but very few letters—most of them not containing any, and scarcely any construction lines are provided. This arrangement makes it much harder to understand the solution, but prevents mechanical reproduction. It puts the difficulty in another place by putting the student squarely on his own feet to provide the details himself. In this way the power of space visualization is greatly enhanced, incidentally giving the student the esthetic satisfaction of doing much of the work himself.

The idea of rotating a polygon about one of its sides and of reflection, even of a stereometric figure, about a line are introduced early and used frequently. In the first two chapters these ideas are treated as distinct problems, without much emphasis of the relations between them; the first time any new idea is introduced it is developed in considerable detail, but when use is made of the same idea later the reader is supposed to be familiar with the principle; one who reads without mastering the details will soon be swamped.

Chapter III, stereometric constructions, begins with a resumé of all the results obtained in the preceding ones, then applies them to a number of constructions of solid geometry. The next chapter, called transformations, while dealing with no ideas that are essentially different from those treated by other authors, gives a very systematic development of the use of rotation about certain lines. The first mention of a profile plane is made on page 62 as an incident in the discussion of transformations. Intersections of prisms, pyramids, etc., roof constructions, excavations complete the ordinary two plane discussion of figures bounded by planes.

Now follows a short chapter on axonometric drawing, together with a comparison with the preceding method, showing its advantages and disadvantages; then comes a chapter on certain skew parallel projections, in particular the military and the cavalier. All three methods are illustrated by a complicated architectural drawing.

The graphical development is here interrupted to give a considerable discussion of the elements of projective geometry, showing various particular kinds of perspective, including affinity and similarity and plane perspective. Cross-ratio and harmonic forms are treated metrically and algebraically. The idea of a curve first appears on page 133—the concepts of tangent, contact, curvature, evolute, and involute are all discussed and the statement explicitly made that our only general method of constructing curves is to locate a number of points and then connect them freehand.

No knowledge of conics is presupposed, but an extensive theory is built up, commencing with Steiner's method of generating a conic as locus of intersection of corresponding rays of two coplanar projective pencils, then establishing the theorems of Pascal, Brianchon, and Desargues, the theory of pole and polar, center and diameters, asymptotes, and focal properties. These ideas are then skilfully applied to a large number of problems of construction.

Curves in space are given a much briefer treatment, details being given only for helices. Then follow developables, cones, cylinders, and curves of intersection, with development, surfaces of revolution, quadric surfaces by means of parallel sections, and finally topographical surfaces, conoids and ruled quadrics. In the forthcoming second volume curves of intersection of curved surfaces and shades and shadows are to be treated.

The first volume of Professor Müller's book was reviewed in the BULLETIN, volume 16, page 136. This first instalment of the second volume maintains the same standard and vigor as the first volume, and furnishes a comprehensive compendium as well as a text-book on the subject.

The volume begins with horizontal projection, and applies it to topography, roof construction, and excavations, the scope and treatment being rather similar to that given by Professor Low. Then follows a chapter on axonometry, with application to the representation of curve surfaces, including a number of metrical problems. An interesting feature is the extensive historical development, given in the form of footnotes.

In this set of four books, one American, and three European, we find a good representation of the relative states of the science, as viewed by the different countries. When shall we be able to regard descriptive geometry as a science coextensive with projective and analytic geometry?

VIRGIL SNYDER.

CORNELL UNIVERSITY,
August, 1913.

SHORTER NOTICES.

Zahlentheorie. By KURT HENSEL. Berlin and Leipzig, G. J. Göschen, 1913. 356+xii pp.

As a knowledge of the elements of the theory of congruence of integers is essential in many branches of mathematics, and as the higher parts of the theory of numbers have enthusiastic devotees, it is not surprising that there are published yearly several books treating the theory of numbers from various standpoints. The usual topics on congruences, including the reciprocity law for quadratic residues, are treated in the present book, but at widely separated intervals, the interspersed material being of quite a different nature described below. Consequently, a reader desirous of acquiring rather quickly a knowledge of the classical theory of congruences will not find the present book so well adapted to his needs as most of the texts available. However, there will be readers who appreciate the opportunity of being able to pick up incidentally this useful information while enjoying