Hensel's new and powerful method for number-theoretic questions, a method analogous to that of power series in the theory of analytic functions. This new method in number theory is that of p-adic numbers described in detail in my extensive review, BULLETIN, volume 17 (1910), pages 23–36, of Hensel's Theorie der algebraischen Zahlen. The newer book treats also of the similar, but more general, g-adic numbers, where g is any integer, whereas p was a prime. The g-adic numbers are, however, compounded in a very simple manner of p_i-adic numbers, where $p_1, p_2, \cdots$ are the prime factors of g, so that the matter finally rests upon the case treated in the former book. The new book is however an essentially new contribution to this subject. It gives also an exposition of series of p-adic numbers and of g-adic numbers; as well as of power series in which the variable and the coefficients are p-adic or g-adic numbers, with a detailed treatment of the exponential and logarithmic functions of a g-adic argument. The final chapter (sixty pages) obtains from the point of view of p-adic numbers the more fundamental properties of binary and ternary quadratic forms.

It is certainly instructive to see the theory of congruences developed by the side of, and largely by means of, the theory of p-adic numbers, a theory shown in Hensel's former book to be a powerful instrument for the investigation of algebraic numbers.

L. E. DICKSON.

Leçons sur les Equations intégrales et les Equations intégro-différentielles. Professées à la Faculté des Sciences de Rome en 1910, par VITO VOLTERRA, et publiées par M. TOMASSETTI et F. S. ZARLATTI. Paris, Gauthier-Villars, 1913. vi+164 pp.

THE subject of integral equations is rapidly coming to the front as one of the most important branches of mathematics. Several books on this subject have been published recently, but the book in question, being the published lectures of one of the founders of the theory of integral equations, will no doubt be received with an especially hearty welcome by mathematicians. As stated in the preface, the book is based on the course of lectures given by Volterra at the University of Rome in 1909–1910. The plan of the lectures, however, has been somewhat changed, partly by adding some interesting investigations of more recent date and partly by leaving out most of the applications. This latter fact seems regrettable, but we are promised by the author a second volume which will be devoted to various applications and also to a more extended presentation of the theory of integro-differential equations.

The object of the author is to give a brief and systematic outline of the entire field of integral equations. The treatment of the topics considered is excellent, and the book will appeal not only to the student who wishes to lay a good foundation for further work but also to the more superficial reader whose object is simply to get some idea of the subject.

The author often confines himself to special cases imposing certain restrictions on the functions with which he is dealing. It might have been desirable to give a more general treatment. In nearly all cases, however, he points out how his methods may be extended to more general cases. Whether this is a good procedure or not is, of course, a matter of opinion. The book covers an unusually large number of topics, and a great number of references are given.

In Chapter I the author gives a brief outline of the theory of functions which depend on all the values of a function in a certain domain, or functions of lines as distinguished from ordinary functions of points. The expression

$$A = \int_a^b \varphi(x)dx$$

is a function of all the values which $\varphi(x)$ takes in the interval (a, b). The form of $\varphi(x)$ determines the variation of A or, in other words, A depends on all the ordinates of the curve $y = \varphi(x)$ in the interval (a, b) and is hence a function of an infinite number of variables. To indicate the fact that a quantity F depends on all the values of a function $\varphi(x)$ in an interval (a, b) the notation

$$F = F|[\varphi_a{}^b(x)]|$$

is used. Starting from this notion the author defines continuity, derivatives, etc., and gives several examples of functions of this kind. The treatment is very brief, and rigorous proofs are omitted, but numerous references are given for the benefit of the reader who wishes to penetrate deeper into this most fascinating field. Confining himself to functions which are developable in a series corresponding to Taylor's series, the author is finally led to the consideration of a general functional equation of which the ordinary linear integral equation is a special case.

In Chapter II the author begins a systematic treatment of integral equations. He first considers two mechanical problems of Abel and Liouville, and then takes up a general discussion of the integral equation of the Volterra type of the second kind, i. e., an equation of the form

$$\varphi(x) = u(x) + \int_0^x K(x, \xi)u(\xi)d\xi,$$

where the kernel $K(x, \xi)$ is supposed to be finite and continuous in the region

$$0 \leqq \xi \leqq x \leqq b.$$

The integral equation is considered as a limiting case for a system of n algebraic equations as n becomes infinite.

The integral equation of the first kind, i. e.,

$$\varphi(x) = \int_0^x K(x, \xi)u(\xi)d\xi,$$

is considered next. Here both K and $\partial K/\partial x$ are supposed to be finite and continuous, and also $\varphi(0) = 0$. A solution is obtained by reducing the given equation to an equation of the second kind. The special case when K becomes infinite of an order $\alpha < 1$ for $x = \xi$ is then discussed. Finally the case when $K(x, x) = 0$ is considered. In discussing this case the elegant method of Lalesco, which consists in making use of linear differential equations, is applied.

The author next considers systems of integral equations and equations containing multiple integrals and shows how the ordinary methods can without serious complications be extended to these cases. A few pages are then devoted to the method of successive approximations as applied to integral equations. Other topics of which a brief discussion is given are generalized equations of the Volterra type, equations where the two limits of integration are variable, and non-linear equations.

In Chapter III equations of the Fredholm type

$$\varphi(x) = u(x) + \lambda \int_0^1 K(x, \xi)u(\xi)d\xi$$

are discussed. The same method of considering the given equation as the limiting case of a system of algebraic equations is used in treating this type. The kernel is supposed to be finite. The case where the kernel becomes infinite is discussed very briefly. A few pages are devoted to systems of equations and equations involving multiple integrals. At the end of the chapter some very interesting applications are made to Dirichlet's problem and to the vibration of strings.

In Chapter IV a very brief and incomplete account of integro-differential equations is given. By such an equation is meant one involving not only the unknown functions under signs of integration but also the unknown functions themselves and their derivatives. No attempt is made to give a systematic treatment of this subject. A few problems from mechanics leading to equations of this kind are discussed. A few pages are also devoted to permutable functions. For a more complete discussion of these very interesting topics the reader is referred to papers by Volterra published in *Acta Mathematica* and *Atti d. R. Accademia dei Lincei.* It is with great pleasure that we receive the news that the author intends to give an exhaustive treatment of these topics in a second volume which will soon be published.

JACOB WESTLUND.

Die komplexen Veränderlichen und ihre Funktionen. Von Dr. GERHARD KOWALEWSKI, Ord. Professor an der Hochschule zu Prag. Leipzig und Berlin, B. G. Teubner, 1911. 455 pp.

THIS volume, by the well-known author of the recently published text on determinants, is intended to be a continuation of the Grundzüge der Differential- und Integralrechnung, which was reviewed on page 531 of volume 19 of the BULLETIN, as well as an introduction to the theory of functions. Some of the very convenient terms introduced in the aforementioned book, such as the expression "fast alle," meaning "all with a finite number of exceptions," so useful in the discussion of propositions involving the limits of sequences, are also used in this book. Other new ones are introduced, as for instance the "Hof" of a point, meaning a circle having the given point as a center. This strikingly descriptive terminology, as well as the interjection in appropriate places