use the term hypothesis in place of theory; and he can certainly not spend too much pains on separating those portions of his work which depend upon such a hypothesis from those which do not.

Chapter IX shows the necessity for an index of prices, and Chapter X, aided by the long mathematical appendix previously reviewed, seeks after the best index. Statistical matter bearing on the previous text now fills two chapters, and the text closes with the discussion of the possibility of keeping the general price level more nearly constant. There are numerous appendices and a very full index.

To the mathematician Fisher's work always appears more sympathetic than that of many of his economic colleagues by virtue of its greater respect for logical and numerical values. This latest book is no exception.

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SHORTER NOTICES.

Vorlesungen über Geometrie, mit besonderer Benutzung der Vorträge von ALFRED CLEBSCH, bearbeitet und herausgegeben von DR. FERDINAND LINDEMANN. Zweite, vermehrte Auflage. Leipzig, Teubner. I Band, I Teil, 1 Lieferung, 1906, vi + 480 pp.; 2 Lieferung, 1910, 288 pp.

THIS revised edition of Clebsch's inimitable and classical lectures is much more than a reprint of the first edition which appeared in 1875–76. Much has been done since then to enrich the subject, and Dr. Lindemann has done well to include the most important of these researches in this revision. The extent of the additions to the text may be seen from a comparison of the number of pages for Part I; this includes the same general subjects in the two editions and yet covers about three times as many pages in the second as in the first (the size of the type and the page differ only slightly). If this ratio is upheld in the other parts, Volume I will contain about 3,000 pages of text.

It is unfortunate that four years should elapse between the two installments. Upon the appearance of the first installment, the completion of Part I was promised within a few months. Even this second installment does not complete Part I, and three years have elapsed since this appeared. At this rate no time limit can be placed upon the appearance of Parts II and III. The custom of sending forth an installment which stops short in the middle of a sentence of some exposition or proof of a theorem is also unfortunate; its macks too much of the serial installments of some thrilling novel, all the more so when one has to wait years instead of merely a day, a week, or a month for the completion of that sentence.

The value of the book is greatly enhanced by the copious footnotes. These give very complete references and brief descriptions of the work done in the subject both before and since Clebsch's time. They also tell, for each chapter, just what parts of the text are taken from the original Clebsch lectures, and which parts have been added for the revised edition.

We shall briefly indicate the more important parts of the text which have been added. Part I is divided into three sections. The first section, entitled Introductory considerations.—Point ranges and pencils of rays, gives the projective analytic geometry of the range and the pencil. Here a few more illustrative examples are inserted and some special cases of general theorems are treated in greater detail. Chapter VII, treating of the general analytic form for projective relations, and Chapter VIII, containing a large number of theorems on medians of quadrilaterals and pentagons, are new.

The second section is entitled The curves of the second order and the second class. Here there have been added six chapters treating respectively of: (VII) the classification of conics according to the form of the projective transformation which generates the same; (XIII) the complete Pascal hexagon with its related Steiner and Kirkman points; (XIV) polar relations and the Pascal hexagon, including the work of Bauer on the ten conics associated with the six points on a conic; (XV) theorems on the application of general analytic methods to conics and quadrilaterals; (XVI) the geometry of the triangle with its Feuerbach circle (following the work of Salmon-Fiedler); (XVII) imaginary elements, giving a deeper insight into the geometric "Sinn" of the imaginary elements according to the beautiful theory of von Staudt (the representation, in the real domain, of two conjugate imaginary points by the involution of which these are the double elements);

and thus all projective results in analytic geometry of the plane in which imaginary elements occur may be interpreted by means of real geometric elements; the exposition in this chapter follows the work of Lüroth and Stolz.

The third section is entitled Introduction to the theory of algebraic forms. Here we find the most extensive additions. The discussion of the identical relations existing between the discriminants of two binary quadratic or cubic forms is extended to the case of n such forms. The other important additions are contained in the last thirteen chapters, none of which had a place in the first edition. These deal respectively with: (X) the representation of binary forms upon a conic, which serves to geometrize and thus classify and emphasize the relations existing among the invariants and covariants of binary forms; (XI) the collineations of a conic into itself and the finite groups of such transformations; (XII) the "tenfold" Brianchon hexagon and the collineations of the related conic; (XIII) polygons and binary forms with their application to the equations of the fifth degree; (XIV) the expansion of binary forms into series (according to the work of Clebsch and Gordan), which serves as a unifying principle between the general methods of the text (using the Clebsch-Aronhold notation) and the non-symbolic and non-homogeneous methods employed to solve special problems; (XV–XVIII) the typical or canonical representation of binary forms by means of partial differential equations for invariants and covariants (according to Clebsch and Gordan), which serves to give an insight into the invariant theory of binary forms of higher order; this canonical representation also gives a method for determining a finite system of invariant forms by means of which all invariants and covariants of a given form may be constructed by rational processes, and finally leads to the proof that the system of invariant forms of a binary algebraic form is finite; (XIX–XXI) the simplest invariants and covariants of a binary form of the fifth and sixth orders, and geometric and arithmetic applications of these to the pentagon and the equation of fifth degree; (XXII) some problems in the theory of binary forms of higher order (only one page of this is printed in the second installment).

Part I is to be completed by a discussion of the representation of binary forms in the complex plane, and an application of this and of cubic forms to the geometry of the triangle and its remarkable points.

Parts II and III are to cover the general theory of algebraic curves, the curves of the third order and the third class, the geometry of an algebraic curve and its relation to the theory of abelian integrals, and a study of the connex.

Dr. Lindemann has done and is doing a monumental work which deserves to rank among the highest in works on geometry; let us hope for its early completion.

JOSEPH LIPKA.

Die Idee der Riemannschen Fläche. Mathematische Vorlesungen an der Universität Göttingen, no. V. By HERMANN WEYL. Leipzig, Teubner, 1913. x + 166 pp. 8 M.

THE volume under review is the fifth of the series of Göttingen Lectures on mathematics, the first four of which were given respectively by Klein, Minkowski, Voigt, and Poincaré. It is announced that four more are under preparation by Runge, Schwarzschild, Toeplitz, and Wiechert.

Before speaking specifically of the work of Weyl, I should like to express my great appreciation of the plan of publishing lectures of this sort, in which the lecturer gives a limited subject mature thought, brings it thoroughly up to date, and perhaps adds contributions of his own. This is a period of what might be called "frenzied" research, in which there is a tendency to rush into print every time a new result is reached. While the intense activity is altogether praiseworthy, it results in a congestion of material in the mathematical journals, and a flood of papers so great that it is impossible for an individual to discover what is important and of interest to him. If the basic ideas of mathematics were shifting as fast as those of some other sciences, the editors of mathematical journals might find themselves in the position of an editor who recently said that his journal was so far behind the material offered it for publication that the authors changed their ideas and wished to withdraw their papers before they could appear in print. Even if we grant that, on the whole, it is for the best that fragmentary investigations in almost unlimited number should continue to be published, the more formal discussions, in which the center of interest is in the subject and not alone in the greater or lesser part which may be new, are of great value and should be encouraged. In this connection it is gratifying that the American Mathematical Society has adopted the policy of publishing the Colloquium Lectures.