

The little book is carefully and clearly written and contains only a few minor errors. On page 58, second line from bottom, read "Nouvelle méthode" instead of "Nouvelles Méthodes." On the next page read J. W. instead of J. J. Stubbs. The reference to J. W. Stubbs and J. R. Ingram is ambiguous. In the *Philosophical Magazine* Stubbs does not make the least reference to Ingram, and evidently claims priority of the discovery.

ARNOLD EMCH.

SHORTER NOTICES.

Poliedri, Curve e Superficie secondo i Metodi della Geometria descrittiva. By GINO LORIA, professor of mathematics at the University of Genoa. Milan, Ulrico Hoepli, 1912. (Manuali Hoepli, nos. 148-149.) xv + 235 pages and 62 figures.

Vorlesungen über darstellende Geometrie. Zweiter Teil: Anwendungen auf ebenflächige Gebilde, Kurven und Flächen. By GINO LORIA. Autorisierte, nach dem italienischen Manuskript bearbeitete, deutsche Ausgabe, by FR. SCHÜTTE, teacher in the gymnasium at Düren. (Teubner's Sammlung, no. XXV₂.) Leipzig and Berlin, B. G. Teubner, 1913. xii + 294 pages and 146 figures.

THE present volume of Professor Loria's treatise presupposes a knowledge of the previous one (*Metodi di Geometria descrittiva*). It is concerned with the applications of the principles there developed to the graphical representation of polyhedra, curves, and surfaces. The style is clear and concise, and each step of a procedure is fully explained, but the theorems of analytical geometry and of the calculus that are made use of are stated without proof or reference, thus providing a series of statements having very questionable value. If the reader is already familiar with these theorems, they need not be repeated here; if he is not familiar with them, the words will convey little, if any, meaning to him.

The first chapter is concerned with the graphical solution of spherical triangles, incidentally including the derivation of the fundamental formulas of spherical trigonometry. The next two chapters discuss the representation of prisms, pyramids, and polyhedra, together with their development and

intersections. The method of Monge is given the preference, but other methods are given in outline. The chapters on plane and on space curves both begin with a summary of definitions, including order, class, tangent, parametric representation, double point, osculating plane, etc. Then follow various empirical methods of constructing tangents and osculating planes. In connection with the method of central projection is found a particularly clear exposition of the relations between a space curve and its plane projection. Plücker's formulas connecting the characteristics of plane curves are presupposed, but no mention is made of the corresponding formulas for space curves. The only curve treated in detail is the cylindrical helix.

The third part, comprising about three fifths of the volume, is devoted to surfaces. As in the preceding cases, the discussion is preceded by a very condensed outline of the theory of surfaces. Three methods of representation are given; that of Monge, the central projection, and that of lines of equal height. It is shown that the first and second both contribute to the study of the apparent contour, but the third method is treated in much greater detail. The well-written chapter on surfaces of revolution employs the method of Monge almost exclusively, and is very similar to the treatment given in the better courses in descriptive geometry in our own technical schools. The discussion of helicoids is rather more analytic than graphic, and is confined to the simpler properties. The chapter on conical and cylindrical surfaces follows the usual development by the method of Monge, though a few interesting applications are made of conical projection.

Of the nine pages on developable surfaces, less than three are concerned with graphical problems; the remainder is properly devoted to a development, not merely an enumeration, of the characteristic properties of these surfaces. The discussion employs synthetic, algebraic, and analytic methods. The last chapter, on non-developable ruled surfaces, gives a briefer outline of the general theory, but proves in detail the correlation of Chasles. After showing that every algebraic ruled surface can be generated by the rectilinear transversals of three directrix curves, the graphical discussion is confined to the case in which one directrix is a straight line, in particular to helicoids and conoids.

The figures are well drawn and the type is excellent; a

reader would find the text easier to follow, however, if greater use had been made of captions, full-faced type, and indentations.

A few typographical errors occur, but most of them will not cause confusion; the only more serious ones are in the foot-note on page 203, the displayed generatrici on page 208, which should read direttrici, and the use of the word incontrano on page 209, line 20, word 6 for toccano.

The German volume is not a translation of the Italian one, although it has the same title and follows in a general way the same development. It is much more extensive, makes more extensive use of algebraic methods, has a larger number of figures, and a larger variety of displayed formulas, theorems, and problems.

The resolution of the problems of trihedra and of spherical triangles is quite similar to the Italian text; the discussion of polyhedra includes the derivation and use of Euler's formulas, and a number of exercises for the reader. But the greatest change is found in connection with the treatment of curves and surfaces. Instead of the embarrassing brevity noted above, we here find a veritable treatise, almost exclusively from the analytic standpoint. The general theory is followed by a description of several algebraic and transcendental curves, in particular the cycloids and spirals. The graphical representation by means of approximations closely follows the Italian text.

In the case of space curves formulas for curvature, torsion, and the relations between a space curve and its plane projection are derived.

The ratio of the material is about the same through the remainder of the volume. It seems odd that it was thought necessary to speak in such detail of quadric surfaces after developing so large a number of formulas for surfaces defined by any analytic function, but the outline of the treatment of the intersection of two quadrics is too brief and too difficult to be of greatest service. The chapter on cones and cylinders skillfully combines the usual graphical representation with algebraic and analytic proofs, thus removing the objection to the more frequent empirical procedure.

Ruled surfaces and developables precede surfaces of revolution; the discussion is more detailed and somewhat more

extensive, yet follows the same line as in the Italian text. The traditional treatment of surfaces of revolution follows the derivation of the general equation and of a number of characteristic properties, including illustrative examples of the ring surface.

The volume closes with a discussion of helicoids, each problem being introduced by a detailed analytic treatment.

No applications to shades and shadows or to other technical uses are made, the authors pointing out that such things would only act as a digression from the purpose of the book, which is to provide a theoretical development, suitable for teachers rather than for practitioners. A generous number of foot-notes give the origin of the important theorems and considerable other interesting information. A third volume is in preparation, which will give a systematic history of the development of the subject, and is to contain a detailed index of all three volumes.

VIRGIL SNYDER.

Vorlesungen über Differential- und Integral-Rechnung. Von EMANUEL CZUBER. Dritte, sorgfältig durchgesehene Auflage. I Band, xiv + 605 pp., II Band, x + 590 pp. 8vo. Leipzig, Teubner, 1912. 12 Mks. each.

It is scarcely necessary to review these well known volumes at great length. Professor Czuber writes in a clear and convincing style and his treatment of the processes of the calculus and the applications is classic.

The second volume of the second edition was reviewed for the BULLETIN by the present writer, May, 1909 (volume 15, pages 392-395). No occasion has since arisen for changing the views there expressed.

The two volumes contain much more material than is ordinarily included in the elementary and advanced courses in the calculus as given in this country. The first volume contains an excellent basis for an elementary course in differential geometry and nearly one third of the second volume is devoted to differential equations and their applications.

The principal changes from the second edition may be briefly noticed. The first volume of the third edition contains a treatment of roulettes, focal lines, or caustic curves in the plane, and loxodromes on surfaces. These curves were not considered in the second edition. The first volume is also