increased in several other less important particulars; e. g., Delaunay's theorem concerning the meridians of surfaces of revolution of constant mean curvature has been added to the articles on the curvature of surfaces.

The second volume of the third edition has been increased and improved principally in the part devoted to differential equations. In particular, the section devoted to the calculus of variations has been entirely rewritten and brought into closer touch with recent work in this subject. Also a section (§ 7) has been added treating of curvilinear integrals and integrals of functions of a complex variable. The section adds but nineteen pages of new material and the treatment is limited to the outlines of the theory.

It is not too much to say, in conclusion, that the two volumes under review form an almost invaluable addition to the library of the teacher of the calculus whether from the point of view of clear and concise statement, or from that of content. It may not be out of place, in this connection, to call attention to the straightforward and rigorous treatment of the fundamental limit

$$\lim_{n \doteq \infty} (1 + 1/n)^n = e$$

in article 30 of the first volume, in comparison with the somewhat apologetic tendency exhibited in some of our modern texts on the calculus to avoid the use of this limit. One may doubt the expediency of presenting all the details of the proof employed by Professor Czuber in a first course in the calculus, but such a doubt scarcely necessitates the use of bizarre, or non-consistent methods.

L. Wayland Dowling.

Lezioni di Geometria proiettiva ed analitica. Di Edgardo Ciani, Professore nella R. Università e nella R. Scuola Navale Superiore di Genova. Pisa, Enrico Spoerri, 1912. v+525 pp.

The plan of replacing the traditional introductory courses in cartesian geometry and in synthetic projective geometry by one set of lectures covering the elements of both subjects is not new to Italian universities. In 1888, through the initiative of Cremona, the faculty of mathematical and physical sciences of the University of Rome sanctioned such a course for the first year students. Texts for these courses are not so plentiful as those presenting only the elements of the classical analytical geometry. There is opportunity for diversity of opinion as to the subject matter, for obviously such a course must omit much. But a greater difficulty is that of presenting the two methods so that the student not only obtains clearly defined ideas of each but also grasps the relations between them and gains some judgment in selecting the method better suited to any particular problem. Perhaps the best known of the texts that have already appeared is that of G. Castelnuovo.*

The work under review was written especially for the engineering students, who constitute the majority of the author's hearers, at the University and the higher naval school of Genoa. One feature that distinguishes it from other texts of its kind is the absence of all mention of homogeneous coordinates. This omission is made deliberately because the author thinks that their use would be premature in the first year of study. He realizes, too, that many well drawn figures and an abundance of carefully selected exercises are necessary for an introductory course. He furnishes two hundred of the former and about three hundred of the latter.

In the first three chapters the author follows the general plan of Castelnuovo (1904–5) and A. del Re† in beginning with a chapter on the fundamental notions of projective geometry and following that by two on the analytical geometry and projectivity for forms of one dimension. The material is well chosen and is presented in a simple and attractive manner.

Plane geometry is dealt with in the next eleven chapters. The first is on cartesian coordinates, the usual equations of the straight line, distance from point to line, etc. Then the author departs from the old order of circle, parabola, ellipse and hyperbola. Instead, he attacks the general conic, beginning with a detailed treatment of its generation by two projective pencils of rays. After calling attention to the ancient Greek conception of these curves, he returns to cartesian coordinates and deduces the analytical representation of the conic. In the next chapter he studies in greater detail the principal analytical properties of the general quadratic equation in two variables and gives their geometrical inter-

* Lezioni die Geometria analitica, Roma, 1903–1909.

† Lezioni di Geometria proiettiva ed analitica, Modena, 1900.

pretation. Then follow chapters on the problem of constructing a conic when certain tangents or points are given; the theory of polarity with reference to a conic; the diameters, axes, center and foci; points common to two conics; the usual higher plane curves; projectivity for forms of two dimensions and projective geometry on a conic.

The book is concluded by eight chapters on solid geometry, —chiefly analytical.

E. B. COWLEY.

Transcendenz von e und π. Ein Beitrag zur höheren Mathematik vom elementaren Standpunkte aus. Von GERHARD HESSENBERG. Leipzig, B. G. Teubner, 1912. x+106 pp.

IT is well known that in approaching the proofs of the transcendence of e and π, either in the original form of Hermite and Lindemann, or in the simplified presentations of Hilbert, Hurwitz, and Gordan, the beginner experiences great difficulty in grasping the significance of such suddenly introduced artifices as the Hermite integral or the Hilbert polynomial.

After some introductory remarks on the "Deus ex machina" appearance of these artifices, the author presents some general reflections, abounding in pedagogical good sense, on "proofs by successive specialization" and "indirect proofs." His point of departure in presenting the proofs—which are in substance those of Hilbert, Hurwitz, and Gordan—is found in the problem of approximating the exponential function by means of the n first partial sums of its power series, each of these sums being weighted in the sense of the method of least squares. In the reviewer's opinion, this mode of presentation should prove natural and plausible to the beginner. All auxiliary propositions (on the rational and exponential functions and on algebraic numbers) are clearly and fully set forth in such a manner as not to obscure with their details the main line of thought in the transcendence proofs, the number-theoretic and analytic features of which are kept well apart.

The preceding developments lead very naturally up to a proof of Lindemann's general theorem on the non-vanishing of a linear aggregate of exponentials with unequal algebraic numbers as exponents and non-vanishing algebraic numbers as coefficients.

This little book is written in a vigorous and pleasing style, as remote from academic dryness as possible without sacrifice