nothing prevents our imagining others." We have however probably said sufficient to show that there is an interest in the book for everyone. JAMES BYRNIE SHAW.

Principes de la Théorie des Fonctions entières d'Ordre infini. By OTTO BLUMENTHAL. Paris, Gauthier-Villars, 1910. vi + 147 pp.

Entire functions of finite order as well as certain classes of entire functions of infinite order have been treated by a number of mathematicians in recent years. In this volume of the Borel series of monographs on the theory of functions, Blumenthal considers the *general* entire function of infinite order, so that the book forms a natural and satisfactory sequel to Borel's own Leçons sur les Fonctions entières of the same series. The interest of the results obtained lies in their generality rather than in their applicability to special entire functions not before treated. These results are in large measure original with Blumenthal although a similar range of ideas had been earlier developed by Kraft (Dissertation, Göttingen, 1903).

It is by the aid of the notion of function-type (fonction-type) that Blumenthal is enabled to overcome the inherent difficulties of the problems which arise. Let $\nu(x)$ be a function which increases to $+\infty$ with x, and let $\mu(x) \geqq \nu(x)$ be a like function whose rate of increase is governed by an inequality

$$\mu(x') \leqq \mu(x)^{1+\epsilon(x)}, \quad x' = x^{1+\frac{1}{\mu(x)^{\epsilon(x)}}}$$

where $\epsilon(x)$ is a decreasing infinitesimal. Then $\mu(x)$ is a *function-type adjoint to* $\nu(x)$ *and* $\epsilon(x)$ if the inequality

$$\mu(x) \leqq \nu(x)^{1+\delta}$$

holds for any $\delta > 0$ and an infinite number of values of x. If $\nu(x)$ is given it is clear that $\mu(x)$ yields a measure of the increase of $\nu(x)$, and at the same time possesses a certain regularity of increase (croissance typique).

The fundamental theorem concerning function-types is that corresponding to any given $\nu(x)$ a function-type $\mu(x)$ adjoint to $\nu(x)$ and some infinitesimal $\epsilon(x)$ may be found. The proof first given by Blumenthal (pages 24–31) contains an error. It is assumed that for any given increasing function $w(x)$ an infinitesimal $\epsilon(x)$ (not the $\epsilon(x)$ of the th orem) can be found such that both $\epsilon(x) \log x$ and $\epsilon(x) \log w(x)$ are increasing functions. But if $\log \log w(x)$ is made to increase by only a finite amount outside of a set of intervals $(x_i, x_i + \delta_i)$, $i = 1, 2, \cdots$, and if these intervals be so chosen that the series

$$\frac{\delta_1}{x_1} + \frac{\delta_2}{x_2} + \cdots$$

converges, it is easy to see that $\epsilon(x)$ cannot approach zero. The second proof (note I, pages 117–128) is however devoid of any objection.

With the aid of the concept of function-type Blumenthal generalizes almost all the results known for entire functions of finite order but it is perhaps not desirable that the reviewer make an outline of this material.

Here then is a book which the mathematician who is interested in the theory of the entire function will find worthy of his attention. GEORGE D. BIRKHOFF.

Les Systèmes d'Equations linéaires a une Infinité d'Inconnues. Par FRÉDÉRIC RIESZ. Paris, Gauthier-Villars, 1913. vi + 182 pages. 6.50 fr.

THIS little book belongs to the collection of monographs on the theory of functions published under the general direction of M. Emile Borel. It deserves the high praise of being pronounced worthy a place in this excellent series.

The purpose of the volume is to give a rapid exposition of the fundamental ideas, of the methods and of the principal results in the theory of linear equations with an infinite number of variables—a theory which is due almost entirely to contemporary mathematicians.

An introduction to the subject is made through a chapter (Chapter I, pages 1–20) devoted to the beginnings of the theory. The method of undetermined coefficients was the first; but this method is not characteristic of the subject. Next comes the work of Fourier, who introduced an important general principle in connection with a special type of problem: naturally, the work of Fourier does not meet the modern requirements of rigor. Mention is also made of the papers of Fürstenau and Kötteritzsch; these are said to have been without importance in the development of the theory. (In this connection see a paper by the reviewer in the *American Journal of Mathematics* for January, 1914.) Finally, an account of