where λ is an odd prime, cannot be true for any three integers x, y, z prime each to each, provided that

(1) no one of these integers is divisible by λ;

(2) one of them is divisible by λ but not by λ^2.

In the discussion of the first of these results there occurs an essential error which has already been pointed out by Mirimanoff (*Comptes rendus*, Paris, 157: 491–492). Fabry's second result was already known to Sophie Germain and Legendre (see Bachmann's Niedere Zahlentheorie, II, page 467).

R. D. CARMICHAEL.

Vectorial Mechanics. By L. SILBERSTEIN. Macmillan and Company, London, 1913. viii + 197 pp.

THERE are not in English so many books on vector analysis and its applications that we may not welcome another. The Gibbs-Wilson is the most extended and detailed as regards vector analysis itself, but contains illustrations from geometry, mechanics, and physics rather than applications to them; it has therefore too much mathematics and too little connected application to be entirely ideal for the young physicist. Coffin's is more evenly balanced, and may serve almost equally well as an introduction to vector analysis and to vector physics. Heaviside's genial treatment is embedded in his Electromagnetic Theory. Now comes Silberstein with a work which passes as lightly as possible over formal vector analysis and concentrates on theoretical mechanics. This is a useful variety to introduce. The notation is that of Heaviside; heavy type for vectors, no sign for the scalar product, and a prefixed V for the vector product.

A number of minor complaints may well be made. The Macmillan zero, more insignificant than an "o," is bad. It is unfortunate to use Clarendons for lettering a figure, especially when italics are used in the text. And what can be the advantage of making the figures (usually) run with their positive direction clockwise as they appear on the page? Why take the velocity potential φ so that $\mathbf{v} = \nabla\varphi$ instead of $\mathbf{v} = -\nabla\varphi$, especially now that Lamb in his classic Hydrodynamics has decided in favor of the latter choice? There are a number of instances which show that the author, despite varied linguistic accomplishments, does not sense the meaning of common English words—otherwise he would not call the component of a vector a scalar, nor would he speak of the algebraic product of the tensors of two vectors after defining the tensor as a positive number. And when he speaks of "that $\Delta\sigma$ of which $d\sigma$ is the limit" he is probably more unfortunate than wrong. Indeed the meaning of his text is seldom in question.

The one very bad mistake we have discovered is the treatment of the curl and of Stokes's theorem (pages 33–4). Silberstein attempts an intrinsic proof by considering

$$\psi = \lim_{\Delta S \doteq 0} \frac{\int_0 \mathbf{f} \bullet d\mathbf{r}}{\Delta S}.$$

The scalar function ψ depends on position x, y, z and on α, β, γ, the direction cosines of the normal to a planar element at x, y, z. He then says that ψ may be written as the component of a vector $\mathbf{C}$ along the normal $\mathbf{n}$, $\psi = \mathbf{C} \bullet \mathbf{n}$. That this does not follow is patent. Indeed if we assume that the limit ψ exists, the only hard thing about Stokes's theorem is precisely to show that ψ may be written as $\mathbf{C} \bullet \mathbf{n}$ with $\mathbf{C}$ dependent on x, y, z but not on α, β, γ. I treated the intrinsic method in an article "Divergence and curl," *American Journal of Science*, volume 23 (1907), pages 214–220, and it has probably been treated by others. We may add that the author uses curl, infinitesimal rotation, and other related ideas so often that before his book is finished he has more than enough material to form a proof of Stokes's theorem.

The first quarter of the Vectorial Mechanics develops the elements of vector analysis, including differentiation and integration, but with no reference to the linear vector function. The student would profit by having at hand a more detailed treatment; but it would be difficult for him to find one where the author seemed so thoroughly to think vectors. The fundamental definition of a vector as a free vector and the relation of the definition to directed quantities in physics is explained fairly well, much better than by Coffin. No distinction is made between axial and polar vector, or between scalars and pseudo-scalars—for which we may be thankful.

Mechanics starts off with d'Alembert's principle, Lagrange's equations, and Hamilton's principle. From this it may be inferred that the reader should have considerable familiarity with mechanics before commencing this book. Next we find the principles of work and energy, center of gravity (momentum), and angular momentum (areas). The treatment of the motion of a rigid body is very pretty. It is, however, necessary to introduce the notion of the symmetric linear function, and unless the reader looks up the reference to Heaviside or similar source, he may find the chapter a trifle hard. The Poinsot motion yields with great ease to vector methods, and the author makes the most of his analysis.

The chapter on the mechanics of deformable bodies begins with a concise summary of properties of the general linear vector function, passes to strains and infinitesimal displacements, discusses surfaces of discontinuity (Hadamard), and terminates with stress. It would be difficult in so short a space (47 pages) to do the work better. There follows thirty pages on hydrodynamics which contain most of the classical theory as far as general properties are concerned, and a few other things. This, too, is thoroughly good.

To this point we have covered only 170 pages of the text. The brevity is partly a matter of conciseness in style, but largely due to systematically thinking and using vectors. Sixty-nine exercises, a table of cartesian-vector equivalents in parallel column, and an index complete the work. We could only wish for fifty pages more in which the classical electromagnetic theory, including a little crystal optics, should be presented as succinctly as the theory of rigid motion, fluid motion, and elastic media. As it is, however, Silberstein has given us an almost ideal introduction to mathematical vector physics.

EDWIN B. WILSON.

NOTES.

THE July number (volume 15, number 3) of the *Transactions of the American Mathematical Society* contains the following papers: "A new principle in the geometry of numbers, with some applications," by H. F. BLICHFELDT; "An application of Severi's theory of a basis to the Kummer and Weddle surfaces," by F. R. SHARPE and C. F. CRAIG; "Transformations of surfaces of Voss," by L. P. EISENHART; "Birational transformations of certain quartic surfaces," by F. R. SHARPE and VIRGIL SNYDER; "One-parameter families of curves in the plane," by G. M. GREEN; "The minimum of a definite integral for unilateral variations in space," by G. A. BLISS and A. L. UNDERHILL; "On a method of comparison for triple-systems," by L. D. CUMMINGS; "An existence theorem