

to advantage in bringing a course in trigonometry in closer touch with one of its main applications.

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*Relativity.* By A. W. CONWAY. Edinburgh Mathematical Tracts, number 3. London, G. Bell and Sons, 1915. 2 s. 43 pp.

THIS tract is a course of four lectures delivered before the Edinburgh Mathematical Colloquium on the subject of relativity. The audience were representative of various branches of science. These four lectures start with fundamentals, followed by a study of the transformation of the electromagnetic equations, applications to radiation and electron theory, and Minkowski's transformation. The lecturer has succeeded very well in presenting the essentials of the relativity hypothesis free from metaphysics, and speculations of any kind. He has a decidedly sane treatment. There are examples enough to make the ideas clear, stated in everyday terms, and not in terms of the usual mathematical model. It is a serviceable introduction.

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*A Theory of Time and Space.* By ALFRED A. ROBB. Cambridge, University Press, 1914. vi + 373 pp.

THIS treatise is an elaboration of a previous publication on the same subject. In brief it is an analysis of space and time relations by means of a single type of order called conical. The author also calls the result optical geometry. The treatment is of an axiomatic character, the few diagrams serving only as schemes. There are twenty-one postulates set down, and from these and various definitions, some two hundred and six theorems are deduced. These ultimately lead to statements which permit an algebraic formulation by the use of four parameters, which may be interpreted as the usual  $x$ ,  $y$ ,  $z$ , and  $t$ , the last having a somewhat different rôle from the others. The notion of relativity of course hovers in the background, but any one seeking light on that notion here will be disappointed, as the book is simply a development of a very abstract geometry of four dimensions.

It is not possible to give a resumé of the contents in a review, but some idea can be gained of the point of view by stating

the fundamental conceptions of the author in a somewhat different and more picturesque manner than he does. The model thus given is of course due to the reviewer and not to the author. Let us surround every point of ordinary three-dimensional space with a sphere, the radii being anything we like, and some radii being considered to be positive, some negative. Then any such sphere with its center may be called an element. We will consider that if all the radii are increased by the same amount the elements are unchanged. Two different spheres about the same center at one time are two distinct elements. It is in this conception that the term conical order becomes appropriate, for if we start with a two-dimensional space and surround the points by circles, when the radii are increased by the same amount we may move the plane of operations parallel to itself a proportional amount, thus generating cones by the expanding circles. This conception however cannot be used in the imagery of three dimensions.

The element  $B$  is said to be *after* the element  $A$  when the distance of the center of  $A$  from that of  $B$  is less than the algebraic difference of the radii of  $B$  and  $A$ , this difference being positive. If the distance between the centers is less than the difference between the radii of  $A$  and  $B$ , this difference being positive,  $B$  is said to be *before*  $A$ . If the distance between centers is greater than the absolute value of the difference of the radii neither  $A$  nor  $B$  is before or after the other. This is the type of order upon which the author bases his whole development. The appropriateness of the terms from an optical point of view is seen if we consider the spheres to be the wave-fronts of light signals from the points. For a signal to proceed from  $A$  to  $B$  within a given time, the element  $B$  must be after the element  $A$ .

The  $\alpha$ -subset of  $A$  consists of all elements whose centers and radii are such that the distances from the center of  $A$  to the other centers are equal to the differences between the radius of  $A$  and the radii of the other elements. The  $\beta$ -subset of  $A$  consists of all elements whose distances from the center of  $A$  equal the differences of their radii and the radius of  $A$ . The element  $A$  belongs to both subsets. If the radii are all increased sufficiently the  $\alpha$ -subset will consist of all elements whose spheres are internally tangent to that of  $A$ , or would become so by further increase of radii.

The elements  $A_1$  and  $A_2$  determine an *optical line* if  $A_2$  is an element of either the  $\alpha$ -subset of  $A_1$  or its  $\beta$ -subset. The optical line then consists of all elements which are in either the two  $\alpha$ -subsets of  $A_1$  and  $A_2$  or their two  $\beta$ -subsets. All these elements will be tangent to the sphere of  $A_1$  at the same point, or will become so when the radii are all sufficiently increased. If  $A_2$  is neither before nor after  $A_1$ , then the two determine a *separation line*, because the elements which have centers on this line may have spheres of such size that no one is before or after any other. In this sense they are like particles on a line in space. If  $A_2$  is not in the  $\alpha$ -subset of  $A_1$  but is after  $A_1$  they determine an *inertia line*, in the sense that a particle properly chosen could move along this line with a given velocity, that is to say elements may be so chosen as to be successively after  $A_1$  and before  $A_2$ . A separation segment is said to have a length  $r$ , which is the second side of a right triangle made on the distance between centers as hypotenuse, and the difference of the radii as first side. An inertia line is said to have a length which is the second side of a right triangle whose hypotenuse is the difference of radii and first side the distance between centers. In the latter case however the unit of measure is taken to be a standard number  $v$  called the velocity of light, and which would be the rate of increase of all the radii of all the elements.

It is clear that the time deduced in such an axiomatic treatment is not the Time of philosophy nor of psychology, but is merely a kind of order. The time involved here does not flow, for the particular system of spheres we chose is a stationary set, and any increase in radii is for convenience merely, since only the differences of the radii are ever considered. Time enters only by constructing a second system of spheres about all the points of space. The new radii are then representative of times (instants) different from those represented by the first set. As the author claims, he is studying a type of order, which permits of the abandonment of the notion of simultaneity save as a local phenomenon. In this sense he is studying relativity. The treatise will be interesting in the main to students of postulational geometries.

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