THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY IN NEW YORK.

THE one hundred and eighty-fourth regular meeting of the Society was held in New York City on Saturday, April 29, 1916. The attendance at the morning and afternoon sessions included the following fifty-one members:

Dr. J. W. Alexander, II, Dr. F. W. Beal, Mr. D. R. Belcher, Dr. A. A. Bennett, Professor E. W. Brown, Professor B. H. Camp, Professor C. W. Cobb, Dr. Emily Coddington, Professor F. N. Cole, Professor J. L. Coolidge, Dr. Lennie P. Copeland, Professor Elizabeth B. Cowley, Professor Louise D. Cummings, Mr. C. H. Currier, Dr. H. B. Curtis, Professor L. P. Eisenhart, Professor H. B. Fine, Dr. C. A. Fischer, Professor C. C. Grove, Dr. Olive C. Hazlett, Professor E. R. Hedrick, Professor E. V. Huntington, Professor Dunham Jackson, Mr. Glenn James, Mr. S. A. Joffe, Professor Edward Kasner, Dr. L. M. Kells, Professor C. J. Keyser, Mr. Harry Langman, Dr. P. H. Linehan, Professor James Maclay, Dr. R. L. Moore, Professor Frank Morley, Mr. G. W. Mullins, Mr. George Paaswell, Professor James Pierpont, Professor H. W. Reddick, Professor R. G. D. Richardson, Mr. J. F. Ritt, Dr. Caroline E. Seely, Dr. H. M. Sheffer, Professor Clara E. Smith, Professor D. E. Smith, Professor P. F. Smith, Professor Oswald Veblen, Mr. J. H. Weaver, Mr. H. E. Webb, Dr. Mary E. Wells, Professor H. S. White, Mr. J. K. Whittemore, Dr. C. E. Wilder.

The President of the Society, Professor E. W. Brown, occupied the chair, being relieved by Vice-President E. R. Hedrick. The Council announced the election of the following persons to membership in the Society: Dr. E. T. Bell, University of Washington; Professor T. R. Eagles, Howard College; Mr. Glenn James, Purdue University; Dr. J. O. Hassler, Chicago, Ill.; Professor G. N. Watson, University College, London; Mr. J. H. Weaver, West Chester, Pa. Six applications for membership in the Society were received.

Professor D. R. Curtiss was reelected a member of the Editorial Committee of the *Transactions*, to serve for three years beginning October 1, 1916. The resignation of Professor

L. E. Dickson as member of the Editorial Committee was accepted to take effect on October 1, 1916, and Professor L. P. Eisenhart was appointed to fill out the remaining year of Professor Dickson's term. Committees were appointed to prepare a list of nominations of officers and other members of the Council to be elected at the annual meeting, and to consider the matter of the publication of the Harvard Colloquium Lectures. A committee was also appointed, consisting of Professors E. V. Huntington, E. B. Wilson, E. H. Moore, R. C. Archibald, and T. H. Gronwall, to consider in cooperation with other scientific bodies the question of the classification of technical literature.

Thirty-seven members and friends assembled at the dinner after the meeting.

The year 1916 marks the twenty-fifth anniversary of the broadening out of the Society into a national organization, and of the founding of the BULLETIN. It is proposed to celebrate this anniversary in an appropriate manner at the coming summer meeting. Some seventy-five of those who joined the Society in or prior to 1891 have retained their membership during these twenty-five years. It is hoped that a large number of these older members may be present at the summer meeting and that a large representation of the younger generation may also be present to take over the responsibility for the Society's progress during the next quarter century.

The following papers were read at this meeting:

(1) Dr. SAMUEL BEATTY: "Derivation of the complementary theorem from the Riemann-Roch theorem."

(2) Mr. J. F. RITT: "The resolution into partial fractions of the reciprocal of an entire function of genus zero."

(3) Mr. J. F. RITT: "Linear differential equations of infinite order with constant coefficients."

(4) Professor C. J. KEYSER: "Concerning autonomous doctrines and doctrinal functions."

(5) Professor EDWARD KASNER: "Element transformations of space for which normal congruences of curves are invariant."

(6) Mr. J. H. WEAVER: "Some extensions of the work of Pappus and Steiner on tangent circles."

(7) Professor J. L. COOLIDGE: "New definitions for Plücker's numbers."

(8) Professor G. C. EVANS: "Integral equations whose

kernels satisfy a certain difference equation in variable differences."

(9) Professor DUNHAM JACKSON: "An elementary boundary value problem."

(10) Professor L. P. EISENHART: "Transformations of conjugate systems with equal point invariants."

(11) Dr. A. A. BENNETT: "An existence theorem for the solution of a type of real mixed difference equation."

(12) Dr. A. A. BENNETT: "A case of iteration in several variables."

(13) Mr. R. W. BRINK: "Some integral tests for the convergence and divergence of infinite series."

(14) Mr. GLENN JAMES: "A theorem on the non-summability of a certain class of series."

(15) Dr. F. J. McMACKIN: "Some theorems in the theory of summable divergent series."

(16) Mr. J. R. KLINE: "A definition of sense on plane curves in non-metrical analysis situs."

(17) Professor H. B. FINE: "On approximations to a solution of a system of numerical equations."

(18) Professor B. H. CAMP: "Fourier multiple integrals."

(19) Dr. G. A. PFEIFFER: "On the conformal mapping of curvilinear angles."

(20) Professor G. C. EVANS: "Proof of Green's theorem by approximating polynomials."

(21) Dr. A. R. SCHWEITZER: "On a type of quasi-transitive functional equations."

(22) Dr. J. W. ALEXANDER, II: "Some generalizations of the Jordan theorem."

(23) Dr. C. E. WILDER: "Expansion problems of ordinary linear differential equations with auxiliary conditions at several points."

(24) Professor E. V. HUNTINGTON: "A simple example of the failure of Duhamel's theorem."

(25) Professor W. F. OSGOOD: "Note on functions of several complex variables."

(26) Professor W. A. WILSON: "On separated sets."

Mr. Brink's paper was communicated to the Society by Professor Birkhoff; Dr. McMackin was introduced by Professor Keyser, and Mr. Kline by Dr. R. L. Moore. The papers of Dr. Beatty, Professor Evans, Mr. Brink, Dr. Pfeiffer, Dr. Schweitzer, Dr. Alexander, Professor Osgood, and Professor Wilson were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. By making use of the idea of complementary bases, without which the complementary theorem cannot be even stated, Dr. Beatty derives the complementary theorem from the less general Riemann-Roch theorem.

2. In Mr. Ritt's first paper, which is auxiliary to the second, are discussed the conditions under which the reciprocal of an entire function $G(z)$, of genus zero, can be resolved into simple elements as if $G(z)$ were a polynomial. If the zeros of $G(z)$ are $a_1, a_2, \cdots, a_n, \cdots$, the resolution is possible when, on and after a certain point,

$$\left|\frac{a_{n+1}}{a_n}\right| > 1 + \frac{k}{n},$$

where $k > 2$.

3. The first part of Mr. Ritt's second paper develops the theory of the operator

$$A = \left(1 - \frac{D}{a_1}\right)\left(1 - \frac{D}{a_2}\right) \cdots \left(1 - \frac{D}{a_n}\right) \cdots,$$

where D denotes differentiation and where

$$\sum_{n=1}^{\infty} \frac{1}{|a_n|}$$

is convergent. This operator, which, strangely enough, does not seem to have been discussed before, has the entire space of analytic functions for its domain of applicability. In the second part is discussed the most general solution of the equation $A\varphi(z) = 0$. The solutions are shown to be uniform and to have no isolated singularities. The analytical representation of the solutions is discussed. There is obtained, incidentally, a known result in the theory of analytic prolongation. In the third part of the paper, which will furnish material for future investigations, more general equations are considered, the principal object being to apply the results to the above equation $A\varphi(z) = 0$.

4. The undefined or primitive terms or elements in a postulationally established theory admit of infinitely many interpretations. Hence such terms are variables. Hence the postulates or primitive so-called propositions are not propositions, being neither true nor false prior to specific interpretation, but are propositional functions, and the same holds of the deduced so-called propositions. Accordingly, the theory constituted by such propositional functions, being neither true nor false prior to assignment of specific values to the primitive variables involved, is not a doctrine but is, Professor Keyser contends, something that may be fitly called a doctrinal function. Just as a propositional function is a source of innumerable propositions, some true, some false, the true ones being called values of the propositional function, so a doctrinal function is a source of innumerable doctrines, some true, some false, arising from diverse specific interpretations of the primitive variables. The true doctrines thus arising ought, naturally, to be called the values of the doctrinal function. It thus appears that Hilbert's so-called "Foundations of Geometry," for example, is, strictly speaking, not a geometry nor any other specific doctrine. It is a 3-dimensional euclidean doctrinal function, of which ordinary euclidean geometry is one value, viz., that value that arises from assigning to the primitive variables (Hilbert's "point," "line," and "plane") such meanings as Hilbert did not assign but Euclid did assign (unfortunately under the caption of "definitions" instead of descriptions). No system of primitive propositional functions, or postulates, is satisfied by a set of specific entities that do not have "excessive meaning," i. e., meaning over and above that required merely to satisfy the system. The various doctrines arising from a given doctrinal function are thus known and distinguished by the differing excessive meanings of the entities involved. For a doctrine to be geometric, part of the excessive meaning of the primitive entities must be the concept of spatial extension.

5. Professor Kasner determines all transformations of lineal elements (x, y, z, y', z') of space such that every normal congruence of curves shall be converted into a normal congruence. The infinite group obtained is isomorphic with the group of contact transformations in space. The only transformations in the new group which convert curves into curves

are the conformal transformations, which form a 10-parameter subgroup. The results are of interest in connection with the optics of general isotropic media.

6. Pappus in Book IV of the Collection develops some properties of infinite systems of tangent circles A which are tangent to two given circles. Mr. Weaver's paper extends these results in two directions.

(1) He finds the analytic expression for the radius of the nth circle in the infinite series, and also the expression for certain other allied lines, and from these expressions develops some infinite series which may be summed geometrically. These series closely resemble those given by Fabry,* from an analytic point of view.

(2) He develops a projective method for constructing conics and investigates some projective properties of tangents and normals to the conics determined by the given circle and the series A. The results here are supplementary to those of Steiner on the same problem,† and include additional extensions in the light of modern projective theory.

7. Professor Coolidge's paper gives new definitions for the order, class, and deficiency of an algebraic plane curve suitable to the case where the curve is required to be real.

8. Professor Evans's first paper appears in full in the present number of the BULLETIN.

9. Professor Jackson's paper appeared in full in the May BULLETIN.

10. Professor Eisenhart considers pairs of surfaces S and S_1, so related that for the congruence of the joins of corresponding points M and M_1, the developables meet S and S_1 in conjugate systems of curves. Let S^{-1} and S' denote the Laplace transforms of S with respect to these conjugate curves, taken as parametric. The tangent planes to S osculate one family of parametric curves on S^{-1} and the other family on S'. In each of these osculating planes there is a pencil of conics tangent to the two curves at the points of osculation. These

* Théorie des Séries à Termes constants, Chap. IV.

† Steiner, Werke, herausgegeben von Weierstrass, vol. I, pp. 47 and ff.

conics determine involutions on the line of intersection of the tangent planes to S and S_1. Involutions are likewise determined on these lines by similar pencils of conics in the tangent planes to S_1. These involutions are the same only in case the parametric conjugate systems on S and S_1 have equal point invariants, in which case S and S_1 are in the relation of a transformation K, previously considered by the author. Certain pencils of quadrics of singular interest may be associated with these pencils of conics. The interrelations of these various configurations are determined. The paper will appear in the *Annals of Mathematics*.

11. In this paper Dr. Bennett points out the fact that a real mixed difference equation of the form

$$y_{h+1}^{(0)} = F[x;\ y_0^{(0)},\ y_0^{(1)},\ \cdots,\ y_0^{(k)};\ y_1^{(0)},\ \cdots,\ y_1^{(k_1)};\ \cdots;\ y_h^{(0)},\ \cdots,\ y_h^{(k_h)}],$$

where by $y_i^{(j)}$ is meant $d^jy(x+i)/dx^j$, will under certain simple and general restrictions have as solutions only functions which are continuous together with all of their derivatives for all non-negative values of x. It is then shown that not only are the independent initial conditions infinite in number, but that solutions may be readily constructed having an infinite number of degrees of freedom, where the Taylor's series of the desired solution is assigned arbitrarily, as divergent series if desired, at each of the points $x = i$, $i = 0, 1, \cdots, h$. Use is made of some of the results obtained by Mr. J. F. Ritt in a forthcoming article in the *Annals of Mathematics*. The present paper, also, will appear in the *Annals*.

12. Dr. Bennett here examines a special case of iteration in several variables. The essential equivalence is pointed out between difference equations in several variables and those functional equations which constitute the subject matter of iteration. In particular, the author examines equations of the form $F_{(i)}[u_1, u_2, \cdots, u_n] \equiv au_i + ((u^2))$, $i = 1, 2, \cdots, m$, where by $((u^2))$ is meant a power series in the m variables $(u_1, u_2, \cdots, u_m)$ for which no term appears of lower than the second degree in the set of m variables together. For the case in which $|a| \neq 0, \neq 1$, an explicit solution is obtained in terms of the integral iterates by a formula which is obtained

from an extension of Newton's interpolation formula. The limiting forms are also discussed. This paper constitutes a partial extension of the general theory of one variable contained in a previous paper by the author, *Annals of Mathematics*, volume 17 (1915). This paper will also appear in the *Annals*.

13. The recurrence formula that defines the general term of an infinite series may be regarded as a difference equation in the partial sum of the series. In Mr. Brink's paper methods are given whereby such a difference equation may be replaced by a differential equation whose solution behaves for large integral values of the variable like the partial sum of the series. In this way the author obtains a sequence of integral tests for the convergence and divergence of series. The Maclaurin-Cauchy test is given with extended conditions as the first of this sequence. Another of the simpler tests presented makes use of the function $r(x)$ which has the property that $r(n)$ is the ratio of u_{n+1} to u_n, u_n being the general term of the series. Under certain restrictions upon $r(x)$ the series converges or diverges with the integral

$$\int_a^\infty e^{\int_a^x \log r(x)dx}\, dx.$$

By means of this and other tests of the sequence a large number of the classical convergence tests are easily established. The tests are readily generalized for multiple series.

14. Mr. James extends the notion of proper divergence to include a certain class of oscillating series. This class contains all series such that for any positive (or negative) C and every M there exists an $m \geqq M$ such that

$$\sum_{p=1}^{n} (S_{m+p} - C) \geqq 0, \ (\text{or } \leqq 0), \ n \geqq m+1.$$

A theorem is established from which it follows that the methods of Borel, Cesàro, LeRoy, Cesàro-Riesz, and others cannot sum series of this class.

15. Hardy and Smail showed that the sufficient condition for the continuity of the sum function of a summable divergent

series of continuous functions is the uniform summability of the series, that is, the uniform convergence of the auxiliary limit, as Borel's integral, Cesàro's mean value, etc. Dr. McMackin shows that this is not a necessary condition for the continuity of the sum, and derives conditions which are both necessary and sufficient. For series which oscillate between finite limits quasi-uniform convergence of the auxiliary limit is shown to be both necessary and sufficient in the case of Borel's integral sum, while in the other case it holds for all summable series with continuous terms. It is also shown that a quasi-uniformly convergent series is quasi-uniformly summable according to the definition of quasi-uniform summability given.

16. Mr. Kline proposes the following definition for sameness of sense on two simple closed curves: The sense $A_1B_1C_1$ on the simple closed curve J_1 and the sense $A_2B_2C_2$ on the simple closed curve J_2 are said to be the same with respect to their common exterior E_{12}, if there exist a simple closed curve J_3 lying entirely in E_{12} and three points A_3, B_3, C_3 on J_3 such that (1) if it is impossible to join A_1 to A_3, B_1 to B_3 and C_1 to C_3 by simple continuous arcs which except for their end points lie entirely in E_{13} (the common exterior of J_1 and J_3) and have no points in common, then it is also impossible to join A_2 to A_3, B_2 to B_3 and C_2 to C_3 by simple continuous arcs lying except for their end points entirely in E_{23} (the common exterior of J_2 and J_3) and having no points in common, or (2) if it is possible to join A_1 to A_3, B_1 to B_3, and C_1 to C_3 in the manner above indicated, then it is also possible to join A_2 to A_3, B_2 to B_3 and C_2 to C_3 as indicated.

It is established that if for one choice of the curve J_3 and of three points A_3, B_3, and C_3 lying thereon, the senses $A_1B_1C_1$ and $A_2B_2C_2$ are the same with respect to E_{12}, then these senses are also the same with respect to any other such choice of J_3 and $A_3B_3C_3$.

It is also shown that if the senses $A_1B_1C_1$ on J_1 and $A_2B_2C_2$ on J_2 are opposite with respect to their common exterior and the senses $A_2B_2C_2$ on J_2 and $A_3B_3C_3$ on J_3 are opposite with respect to their common exterior, then the senses $A_1B_1C_1$ on J_1 and $A_3B_3C_3$ on J_3 are the same with respect to their common exterior.

17. Professor Fine's paper is concerned with the proof of the following theorem: Let $f_i(x_1, x_2, \cdots, x_n)$, $i = 1, 2, \cdots, n$, be real functions of the real variables $x_1, x_2, \cdots, x_n$ which have continuous first and second derivatives in the region under consideration. Let $(x_1^0, x_2^0, \cdots, x_n^0)$ belong to this region and let $\xi_1, \xi_2, \cdots, \xi_n$ be the numbers defined by the equations

$$f_i(x_1^0, x_2^0, \cdots, x_n^0) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^0} \xi_k = 0 \quad (i = 1, 2, \cdots, n).$$

Again let S denote the circle, sphere, or hypersphere whose center is $(x_1^0 + \xi_1, x_2^0 + \xi_2, \cdots, x_n^0 + \xi_n)$ and whose radius is $\rho = [\Sigma \xi_k^2]^{\frac{1}{2}}$, and suppose that, for this region S, $D > 0$ denotes the lower bound of the absolute values of the functional determinant of the functions f_i, $M < \infty$ the upper bound of the absolute values of the first minors of this determinant, and $N < \infty$ the upper bound of the absolute values of the second derivatives of the functions f_i.

Then if

$$\left[\sum_{i=1}^{n} f_i^2(x_1^0, x_2^0, \cdots, x_n^0) \right]^{\frac{1}{2}} < \frac{D^2}{n^3 \sqrt{n} M^2 N},$$

the system of equations $f_i(x_1, x_2, \cdots, x_n) = 0$ has one and but one solution in S, and this solution may be approximated to uninterruptedly by successive determinations of $\xi_1^{(j)}, \xi_2^{(j)}, \cdots, \xi_n^{(j)}$ and $x_1^{(j+1)}, x_2^{(j+1)}, \cdots, x_n^{(j+1)}$ by the formulas

$$f_i(x_1^{(j)}, x_2^{(j)}, \cdots, x_n^{(j)}) + \sum_{k=1}^{n} \frac{\partial f_i}{\partial x_k^{(j)}} \xi_k^{(j)} = 0,$$

$$x_i^{(j+1)} = x_i^{(j)} + \xi_i^{(j)},$$

the solution being $\lim_{j \to \infty} (x_1^{(j)}, x_2^{(j)}, \cdots, x_n^{(j)})$.

18. By using G. H. Hardy's definition of a non-absolutely convergent multiple integral it is possible to show that an arbitrary function may be expressed as a Fourier double integral under circumstances much more general than is possible when the iterated integral is used. The results are useful in connection with the Fourier quadruple integral which appears in three dimensional physical problems. Professor Camp's paper also considers the continuity, differentiability,

and integrability with respect to parameters of multiple integrals over infinite fields.

19. In a previous paper Dr. Pfeiffer has shown that divergent power series may be formally obtained in seeking a conformal transformation which maps a curvilinear angle upon a rectilinear angle of the same magnitude and which is analytic about the vertex of the angle. In the present paper he shows that there exist functions $f(z)$ which map the interior of the curvilinear angle upon the interior of a rectilinear angle of the same magnitude conformally such that the sum of the first n terms of the power series already referred to represents the function $f(z)$ asymptotically up to order n. The mapping defined by the function $f(z)$ is conformal on the sides of the angle, except of course at the vertex, where it is continuous.

The above refers to angles whose magnitudes are not commensurable with π, for in the contrary case the writer has already shown that all transformations (analytic about the vertex) obtained formally are convergent.

20. Many problems in mathematical physics are expressed in terms of integral relations, and it is only by passing to a limit that the differential equations are secured. There may be nothing in the physical problem which corresponds to the limit. An analysis is therefore desirable which involves rather the integral than the differential relations. Several proofs have been given of Green's theorem on this basis. Professor Evans notes a new proof, possible by means of an expansion in polynomials, of the same nature as the proof oı a theorem in de la Vallée Poussin's Cours d'Analyse, volume II, page 24, edition of 1912.

21. Let $\lambda_i(x)$ $(i = 0, 1, 2, \cdots, n+1)$ be functions of a single variable; then Dr. Schweitzer defines the type of quasi-transitive functional equations

$$f\{\lambda_1 u_1, \lambda_2 u_2, \cdots, \lambda_{n+1} u_{n+1}\} = \lambda_0 f\{\phi_1, \phi_2, \cdots, \phi_{n+1}\},$$

where $u_j = f(t_1, t_2, \cdots, t_n, x_j)$, $j = 1, 2, \cdots, n+1$, and ϕ_j denote functions of the variables $x_1, x_2, \cdots, x_{n+1}$. In the present paper the special case $\phi_j = \sum_{k=1}^{n+1} m_{jk} x_k$ only is considered. Then the resulting class of functional equations may be put into (1, 1) correspondence with the class of linear homogeneous substitutions and one readily defines, at least formally, a representation of an arbitrary finite group of such substitutions. On the other hand, if $\lambda_i(x) = x$ $(i = 0, 1, 2, \cdots, n+1)$ and if $f\{x_1 + y_1, \cdots, x_{n+1} + y_{n+1}\} = f(x_1, \cdots, x_{n+1}) + f(y_1, \cdots, y_{n+1})$, i. e., if $f(x_1, x_2, \cdots, x_{n+1}) = \sum_{j=1}^{n+1} \xi_j \cdot x_j$, then one obtains $n+2$ equations conditioning the constants ξ_j. By rational elimination, it is found that each of the constants $\xi_2, \xi_3, \cdots, \xi_{n+1}$ satisfies an algebraic equation of the nth degree with coefficients which are rational functions of the constants $M_{st} = m_{st} - m_{1t}$ $(s, t = 1, 2, 3, \cdots, n+1; s_1 \neq 1)$. In particular, ξ_{n+1} is a root of the characteristic equation of a certain linear homogeneous substitution.

22. Dr. Alexander's paper contains a simplified proof of Jordan's theorem that a simply closed curve subdivides the plane into two and only two regions. By generalizing the method employed, theorems analogous to Jordan's theorem about k-dimensional manifolds in n-dimensional space are proved.

23. In a paper in the *Transactions* for 1908 Birkhoff considers certain problems connected with the differential system consisting of the equation

$$\frac{d^n u}{dx^n} + * + p_2 \frac{d^{n-2} u}{dx^{n-2}} + \cdots + p_n u + \lambda u = 0$$

and n boundary conditions $W_i(u) = 0$, in which the W's are linear combinations of the values of u and its first $n-1$ derivatives at the end points of the interval over which the system is considered. Among other things he defines a Green's function, $G(x, s, \lambda)$, for this system and proves that for a system with "regular" boundary conditions

$$\lim_{m=\infty} \frac{1}{2\pi i} \int_\Gamma \int_a^b G(x, s, \lambda) f(s) ds d\lambda = f(x),$$

for any function $f(x)$ possessing a continuous derivative, where Γ is a contour in the λ plane enclosing the first m poles of $G(x, s, \lambda)$.

In the present paper Dr. Wilder extends this result in that he adds to the W's linear combinations of the values of u and its first $n - 1$ derivatives at any finite number of points interior to the interval. The Green's function for this system is then defined by the same formula as is used by Birkhoff and it is found that the above integral converges to $f(x)$ provided $f(x)$ has a certain number of derivatives, which number never need exceed n, and provided certain determinants formed from the constants of the auxiliary conditions do not vanish. In the case n is even it is further necessary to assume that the second point from either end of the interval is farther from that end than the first point from the other end is from that end.

24. Professor Huntington's paper refers to the theorem of Duhamel already discussed by Osgood, R. L. Moore, and Bliss in the *Annals of Mathematics* for 1903, 1912, and 1914, namely: If $\alpha_1, \alpha_2, \cdots, \alpha_n$ and $\beta_1, \beta_2, \cdots, \beta_n$ are two sets of infinitesimals such that $\lim_{n=\infty} (\beta_i/\alpha_i) = 1$; and if $\lim_{n=\infty} [\alpha_1 + \alpha_2 + \cdots + \alpha_n] = a$ exists, then $\lim_{n=\infty} [\beta_1 + \beta_2 + \cdots + \beta_n]$ will also exist, and be equal to a. The following example of the failure of this theorem is simpler than examples that have been previously given: Let $\alpha_i = a/n$, and $\beta_i = a/n + 2ic/n^2$, where a and c are fixed constants. Then $\lim \beta_i/\alpha_i = 1$ as $n = \infty$; but $\lim \Sigma\alpha_i = a$, while $\lim \Sigma\beta_i = a + c$.

25. Professor Osgood's paper appeared in full in the June BULLETIN.

26. Professor Wilson's paper appeared in full in the May BULLETIN.

F. N. COLE,
Secretary.

APPLICATION OF AN EQUATION IN VARIABLE DIFFERENCES TO INTEGRAL EQUATIONS.

BY PROFESSOR G. C. EVANS.

(Read before the American Mathematical Society, April 29, 1916.)

IT is known that if the kernel of an integral equation of Volterra type is in the simple form of the difference alone of the