SHORTER NOTICES.

Projective Geometry. By L. WAYLAND DOWLING. McGraw Hill Book Company, 1917. xiii + 215 pp.

THIS book (as the preface tells us) has been developed from a course of lectures given by the author for a number of years at the University of Wisconsin.

In its treatment of the subject it follows closely the main lines of the classical text of Reye in nomenclature and in point of view. It begins with an introductory chapter giving definitions of the elements, projection, section, and ideal elements. The point range, sheaf of lines, sheaf of planes, field of points, field of lines, bundle of lines, bundle of planes, space of points, space of planes, special linear complex, and space of lines are named as the eleven primitive forms. No formal definition of primitive form is given which would enable the reader to judge why exactly these eleven forms are given and others are excluded.

The concept of motion of a line about a point and that of parallelism are made use of in defining an ideal point, and this is followed by the "fundamental assumption: On every straight line there is one and only one ideal or infinitely distant point. This point makes the line continuous from any one point on it to any other point on it in either direction. Through a given point there can be drawn one and only one line parallel to a given line. This parallel intersects the given line in the ideal or infinitely distant point."

This quotation perhaps shows the point of view of the author in basing his projective geometry on a completely developed euclidean geometry. Duality is introduced by means of illustrations followed by a statement of the method of obtaining new theorems from others by interchanging elements and their reciprocals in the statements. Neither the logical bearing of the principle nor the extent of its validity is discussed. The study of the complete quadrangle leads up to harmonic forms and the cross-ratio of a harmonic range is shown to be -1 by the use of ordinary metrics. From harmonic ranges harmonic scales are developed. The harmonic separation theorems are obtained from a free and intuitive use of continuity. A form of the Dedekind postulate is

introduced using the concepts of segments and order as modified by the introduction of ideal elements. The proof of the fundamental theorem of von Staudt is then based on continuity. Curves and envelopes of the second order, poles and polars with respect to a curve are then studied in considerable detail, with the use of metrics for some properties.

An involution on a form is defined as a cyclic projectivity of order 2 and conjugate imaginary points, lines and planes are introduced from the elliptic involution. An entire chapter is devoted to the focal properties of conics. Collineations, dualities, affinities, and polarities are treated briefly.

The figures are fairly well done, the typography is very good, and the volume as a whole is neat and attractive. The large number of exercises scattered throughout the book add much to its utility as a textbook. The little historical notes although very brief are also valuable and stimulating.

In the opinion of the reviewer, such a book as this one, which is avowedly not concerned with the more logical phases of the subject, would be more valuable to many if it gave at least references to such books as that of Veblen and Young, where such treatment could be found. On the whole, however, the book will no doubt be of much service in beginning courses in the subject.

F. W. OWENS.

Introduction to the Elementary Functions. By RAYMOND BENEDICT MCCLENON, with the editorial cooperation of WILLIAM JAMES RUSK. Boston, Ginn and Company, 1918. x + 244 pp.

UNDER the above title the authors present in book form their idea of the content of a required course in mathematics for freshmen in a small college. As presented, the text is the result of five years of teaching of the course in such a college. The elementary functions are those from plane trigonometry, plane analytic geometry, and the elements of differential calculus. These subjects are combined with such review topics from algebra as is necessary to bind the topics presented into an organic whole. The chapters are not sections taken from a single one of the principal subjects, but are mixtures. The calculus covers only the differentiation of algebraic functions with applications to rate and maxima and minima problems.