THE THIRTEENTH REGULAR MEETING OF THE SOUTHWESTERN SECTION.

THE thirteenth regular meeting of the Southwestern Section of the American Mathematical Society was held at the University of Nebraska at Lincoln, on Saturday, November 27, 1920. About twenty persons attended the meeting, including the following members of the Society:

Professor C. H. Ashton, Professor W. C. Brenke, Professor A. L. Candy, Professor C. A. Epperson, Professor M. G. Gaba, Professor E. R. Hedrick, Professor Louis Ingold, Professor T. A. Pierce, Professor H. L. Rietz, Professor W. H. Roever, Professor Oscar Schmiedel, and Professor E. B. Stouffer.

On Friday evening, before the meeting, a smoker was given for the visiting members and their friends, in the University Temple. At this gathering, Professor Hedrick spoke informally concerning the reports of the National Committee on Mathematical Requirements. An interesting discussion followed this informal address. A feature much appreciated by those present was the exhibit of rare mathematical manuscripts and textbooks, by Professor T. J. Fitzpatrick.

At the business session it was decided to hold the next meeting of the Section at the University of Missouri at Columbia. The following committee was appointed:
Professor E. R. Hedrick (Chairman), Professor W. C. Brenke, and Professor E. B. Stouffer (Secretary).

The following papers were presented:

(1) Professor H. J. ETTLINGER: "Existence and oscillation theorems for a system of n differential equations of the second order."

(2) Professor OTTO DUNKEL: "The curve which with its caustic encloses the minimum area."

(3) Professors E. R. HEDRICK, LOUIS INGOLD, and W. D. A. WESTFALL: "Classification of infinities of transformations."

(4) Professor W. C. BRENKE: "On the convergence of certain types of infinite determinants."

(5) Professor LOUIS INGOLD: "Rotation formulas and invariants."

(6) Professor T. A. PIERCE: "Note on Bernoulli's numbers."

(7) Professor OSCAR SCHMIEDEL: "Summation of certain infinite series in finite form. Preliminary report."

(8) Professor M. G. GABA: "A set of postulates for line geometry in terms of line and transformation."

(9) Professor E. B. STOUFFER: "Calculation of the invariants and covariants for ruled surfaces."

(10) Professor E. B. STOUFFER: "Semi-covariants of a general system of linear homogeneous differential equations."

(11) Professor H. L. RIETZ: "Frequency distributions that result from appyling certain transformations to normally distributed variates."

(12) Professor S. LEFSCHETZ: "On certain invariant numbers of algebraic varieties, with application to abelian varieties."

In the absence of the authors, the papers of Professor Ettlinger, Professor Dunkel, and Professor Lefschetz were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. Professor Ettlinger considers the system

$$\frac{d}{dx_i}\left[K_i(x_i, \lambda_1, \lambda_2, \cdots, \lambda_n)\frac{du}{dx_i}\right] - G_i(x_i, \lambda_1, \lambda_2, \cdots, \lambda_n)u = 0$$

$$A_{ij}u_i(a_i) - B_{ij}K_i(a_i, \lambda_1, \lambda_2, \cdots, \lambda_n)\frac{du_i(a_i)}{dx_i}$$

$$\equiv C_{ij}u_i(b_i) - D_{ij}K_i(b_i, \lambda_1, \lambda_2, \cdots, \lambda_n)\frac{du_i(b_i)}{dx_i},$$

where A_{ij}, B_{ij}, C_{ij}, D_{ij} are functions of $\lambda_1, \lambda_2, \cdots, \lambda_n$, and where $i = 1, 2, \cdots n$ and $j = 1, 2$. Under suitable restrictions, the existence of characteristic numbers for the system is proved, and an oscillation theorem established.

2. Necessary conditions for the minimum area between a curve and its caustic were given by Professor P. R. Rider in a paper read at the April meeting of the Chicago Section of the Society. In this paper Professor Dunkel derives conditions for the minimum area which are necessary and sufficient. The properties of the minimizing curve and its caustic are studied, and it is shown that in the general case the first has a single cusp while the second has two, and that the three cusps have parallel tangents. Several cases of end conditions are examined.

3. In this paper Professors Hedrick, Westfall and Ingold continue the investigation of transformations of which an earlier portion was given in a paper at the summer meeting of the Society in Chicago. A classification of the infinities of a transformation is made, and their principal characteristics are discussed, in a manner analogous to the corresponding matter in the theory of functions of a complex variable.

4. It can be shown easily that the infinite determinant $D(a)$ converges to the value zero if the absolute values of its elements a_{ik} form a convergent series. Professor Brenke shows that this remains true if the elements have the form $\lambda + a_{ik}$, where λ is a constant and $\Sigma|a_{ik}|$ converges; also that the infinite secular determinant, whose principal diagonal elements are $x + a_{ii}$, converges for $|x| \leqq 1$, the value being zero for $|x| < 1$.

5. In this paper Professor Ingold extends the application of the formulas of a former paper on "Rotations in a function space" and shows that certain of these formulas give rise to important invariants. The case of the rotations of a system of two vectors is treated in considerable detail.

6. In his note Professor Pierce sets up an explicit formula for the nth Bernoulli number and uses this formula for determining certain factors of the denominators of Bernoulli's numbers.

7. In this paper Professor Schmiedel deals with series principally of the type $1 + 2^a x + 3^a x^2 + \cdots + (m + 1)^a x^m$, and seeks to find expressions for the sum valid for all integral values of a and m. Two expressions for each sum are found according as m is positive or negative, when one or the other of the series is convergent. The special case $x = 1$ is not excepted.

Consideration of the sum for a negative being withheld, the paper is given as a preliminary report.

8. Professor Gaba takes for his basis an undefined element called line and an undefined operation on lines called transformation. In terms of line and transformation point and plane are defined. His six postulates, which are theorems of

line geometry, are sufficient to prove as theorems the postulates of Hedrick and Ingold or of Veblen and Young. Independence examples for the postulates are given.

9. In Wilczynski's Projective Differential Geometry of Curves and Ruled Surfaces, calculation is made of a system of invariants and covariants associated with non-developable ruled surfaces. In the present paper, Professor Stouffer obtains the same results by methods involving much less labor. The general system of two linear homogeneous differential equations of the second order is first reduced to a canonical form, the invariants and covariants of this simplified form are then calculated, and the results thus obtained are finally expressed in terms of the coefficients and variables of the original system of differential equations.

10. In a paper read before the Society in April, 1920 (see abstract in this BULLETIN for June, 1920) Professor Stouffer calculated a complete system of seminvariants of a general system of linear homogeneous differential equations of the mth order in n variables. Those results are now extended by the calculation of a complete system of semi-covariants of the same general system of differential equations.

11. In this paper, Professor Rietz gives the results of an investigation into the nature of the frequency distributions that result from applying certain transformations to each variate of a normal distribution. The transformations used are suggested by experience with various systems of variates, some of which are measurements of distances, others of surfaces, and others of volumes. In particular, if the transformation replaces each variate x by a variate $z = kx^n$, where k and n are constants, the frequency function that gives the distribution of z's is found, and some of its interesting properties are established for special values of n.

12. Professor Lefschetz's paper will appear in an early number of the *Transactions*.

LOUIS INGOLD,
Acting Secretary of the Section.