lar foundation which enables us to publish a series of works of this nature at a nominal price, as is done in several European countries. No better field than this is today open in this country for establishing a relatively small foundation which should seek to satisfy a hunger for good reading. With the work of this British society in mind, one can readily excuse the lack of an index, and the poor paper which war conditions have imposed.

DAVID EUGENE SMITH.

The Theory of the Imaginary in Geometry together with the Trigonometry of the Imaginary. By J. L. S. HATTON, principal and professor of mathematics, East London College. Cambridge, England, University Press, 1920. 216 pp. and 96 figures.

THE word theory in the above title is to be understood in a very non-technical sense. Indeed, apart from the idea of the invariant elements of an elliptic involution on a straight line, no theory is found at all. The purpose of the book is rather to furnish a certain graphical representation of imaginaries under a number of conventions more or less well known. Three concepts run through the work: first, an incompletely defined idea of the nature of an imaginary; second, the analogy with the geometry of reals; third, the use of coordinate methods, assuming the algebra of imaginaries.

Given a real point O and a real constant k, an imaginary point P is defined by the equation $OP^2 = -k^2$. The two imaginary points P and P' are the double points of an involution having O for center, and ik for parameter. The algebra of imaginaries is now assumed, and a geometry of imaginary distances on a straight line is built upon it. The reader is repeatedly reminded that in themselves there is no difference between real and imaginary points; that differences exist solely in their relations to other points. In the extension to two dimensions both x and ix are plotted on a horizontal line, while y and iy are plotted on a vertical line. Imaginary lines are dotted, and points having one or both coordinates imaginary are enclosed by parentheses, but otherwise the same figures are used for proofs, either by the methods of elementary geometry, or by coordinate methods.

In the algebra of segments it is shown that an imaginary distance $O'D'$ can be expressed in the form iOD, wherein OD is a real segment, or at most by OD times some number. Now follows a long development of the extension of cross ratios, etc., to imaginaries. In fact every word of this is found implicitly in any treatment of the invariance of cross ratios under linear fractional transformation.

In Chapter II the conic with a real branch is introduced, beginning with involutions of conjugate points on lines having imaginary points on the conic. If the coefficients in the equation of a circle are real, the usual graph of $x^2 + y^2 = a^2$ for real x and real y is followed by replacing y by iy, then proceeding as before. The former locus is called the (1, 1) branch, and the latter the (1, i) branch of the circle. Similarly, it has a (i, 1) branch, and another, (i, i), but the latter has no graph. This idea is applied in all detail to ellipses, hyperbolas, and parabolas; in the case of the central conics it is also followed by replacing rectangular coordinates by a pair of conjugate diameters. The ordinary theorems of poles and polars, and the theorems of Pascal, Brianchon, Desargues, Carnot are shown to apply. Indeed, after having established the applicability of cross ratios in the earlier chapters, all these proofs can be applied in the same manner as to reals, without changing a word.

Imaginary angles are brought in in Chapter III. The reasoning by analogy has no meaning when applied to minimal lines. It is not shown why the results should be definite and unique in all other cases, but the statements on pages 71 and 73 are restricted by an undefined "in general." On page 73 it is tacitly assumed that the angle between the two bisectors of an imaginary angle is a right angle, and it is stated as a theorem later on that the sum of all the imaginary angles on one side of a straight line, measured at a point on the line, is two right angles. An imaginary line may rotate about a real line from a position of coincidence to one of perpendicularity with the real line. "A right angle may be divided into a series of imaginary angles." On page 70 we are told that "the point at infinity on the line is of the same nature as the base point. It can be regarded as real or imaginary." On page 91 we find: "the sine of θi increases from 0 to $i\infty$." After some skirmishing, trigonometry of imaginary angles is treated by the usual exponential formulas.

A curious mixture of premises introduces the critical lines, that is, those passing through the circle points. It is by no means clear that these are the only exceptions to be made in the application of the preceding theory.

The most satisfactory part of the entire treatise is that founded on the analytic basis. Here the reason for the exceptional nature of the critical lines becomes at once evident. The book could have been materially improved if the direct contradiction met in attempting to measure either distances or angles concerned with these lines had been pointed out.

Notwithstanding the preceding remarks, the book under review is not without merit. It is plainly and consistently written, the development frequently involves considerable skill, the results include a large number that are distinctly worth while, and the exercises furnish mental gymnastic material for many a lesson. The reviewer must contend, however, that it does not furnisb a theory of imaginaries.

VIRGIL SNYDER.

COMMENT ON A PREVIOUS REVIEW.

In Professor Wilson's review of Eddington's *Space, Time and Gravitation,* which appeared in a recent number of this BULLETIN, he expresses his doubt as to the accuracy of Eddington's statement that Riemann never dreampt of a physical application of his analysis. That this doubt is amply justified and that Eddington's statement shows a lack of acquaintance with certain portions of the historical background of the general relativity theory, may be readily shown by a brief quotation from Riemann's famous Habilitationsschrift *Uber die Hypothesen welche dcr Geometrie zu Grunde liegen.* (We make use of Clifford's translation.)

"Either, therefore, the reality which underlies space must form a discrete manifoldness, or we must seek the ground of its metric relations outside it, in binding forces which act upon it. The answers to these questions can only be got by starting from the conception of phenomena which has hitherto been justified by experience, and which Newton assumed as a foundation, and by making in this conception the successive changes required by facts which it cannot explain. . . . This leads us into the domain of another science, of physics, into which the object of this work does not allow us to go to-day."

This statement (made in 1854) foreshadows the work of Einstein, and thus furnishes further evidence, if further evidence be deemed necessary, of the remarkable insight into mathematical and physical questions possessed by Riemann.

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