SHORTER NOTICES

Lezione di Geometria Proiettiva. Federico Amodeo. Third edition, second reprint. Naples, Luigi Pierro, 1920. 450 pp.

The second reprint of the third edition of this work contains an appendix (43 pp.) of modifications and additions made since the first appearance of the third edition (1905).

The text consists of two parts and each part of three chapters. There is an introductory chapter (66 pp.) preceding Part I in which is set forth the fundamental postulates: definitions of primitive forms; the notions of perspectivity, projectivity, homography, correlation, etc.; the theorem of Desargues and its consequences.

The fundamental postulates are those formulated by Amodeo in ATTI DELL'ACCADEMIA DELLE SCIENZE DI TORINO, March, 1891. The first five deal with the existence of elements, denoted by S_i, $i=0, 1, 2, 3$. The sixth postulate asserts the closure of S_1, considered as a class of S_0's, and thus provides for an ideal point on each line (when $S_0 \equiv$ point, $S_1 \equiv$ line). Postulate seven asserts the invariance of the property of separation of two couples of elements on a primitive form under projection and section. There are two more postulates contained in Chapter I of Part I. The first is equivalent to the Postulate of Archimedes from which is deduced the existence of an infinity of elements of a "succession" (previously defined by harmonic groups) between any two elements of the succession; and the second is the Dedekind Postulate from which is deduced the continuity of each primitive form.

Part I, 217 pages, is devoted to the projective geometry of "simple forms" (primitive forms). It contains Von Staudt's Theorem and its consequences; the theory of projectivities upon primitive forms, of involutions, of polarities in S_2 and S_3; the definition of imaginary elements; etc. Polarities in euclidean space (S_2 and S_3) have centers, diameters, axes, foci, but no explicit mention is here made of conics or cones.

Imaginary elements on any primitive form are defined as double points of an elliptic projectivity upon the form, and are represented by four elements $A\ B\ C\ D$ such that A corresponds to B, B to C, and C to D ($A\ D$ not separated by $B\ C$). This method was given by Amodeo in lectures (lithographed), 1887–88, on homographies in the binary field, and printed in GIORNALE DI BATTAGLINI, volume 26, 1888, in a paper entitled *Fasci di omographie e rappresentazione geometrica degli elementi imaginarii.* Professor J. Rey Pastor, in his book *Fundamentos de la Geometria Proyectiva Superior* (1916), calls this "the method of Amodeo." But the method apparently differs very little from that of Von Staudt, since a pair of conjugate imaginary elements defined as above is also defined by a unique elliptic involution; namely the double-points involution (Involuzione unita, Doppelpunktsinvolution).

Part II, 165 pp., is devoted to forms of the second order and one dimension. These forms are defined by means of projectively related

primitive forms rather than as loci of corresponding incident elements in polarities, or as loci corresponding to circles in perspectively related forms, thus following the road chosen by Reye rather than that selected by Von Staudt, or Steiner and Cremona.

The book as a whole is full of detail with good sets of exercises after each chapter. The presentation is clear. The method is synthetic for the most part, although the author does not hesitate to use projective coordinates and the resulting analysis somewhat freely throughout the book. For students who wish to pursue geometry beyond the ordinary undergraduate curriculum in our American universities, Professor Amodeo's book will be both suggestive and inspiring.

L. WAYLAND DOWLING

United States Life Tables 1890, 1901, 1910 and 1901–10. Explanatory Text, Mathematical Theory, Computations, Graphs, and Original Statistics. Also Tables of Life Annuities, Life Tables of Foreign Countries, Mortality Tables of Life Insurance Companies. By James W. Glover. Washington, Government Printing Office, 1921. 496 pp.

The preparation of this volume marks an important advance in the study of vital statistics in the United States.

The work is divided into eight parts. Part I gives a nontechnical description and explanation of life table functions. Part II gives 74 mortality tables based on different subclasses of the population of registration states. Part III gives twelve life tables of foreign countries and ten mortality tables based on experiences of life insurance companies. Part IV presents graphs of life table functions. Part V gives certain life annuities, life insurance premiums and commutation columns. Part VI gives the mathematical theory of the construction of life tables. Part VII gives a detailed account of the process of carrying out the calculations. Part VIII gives tables of the original statistics and an explanation of the different types of data.

The many tables presented in this volume are simply a mine of information for comparative purposes. Part VI on the mathematical theory of the construction of life tables is the part of the work which should prove of much interest to the actuaries interested in the fundamental theory of life table construction. The reviewer believes it correct to say that pages 329–344 give the first clear and detailed statement in the English language of the theory of the continuous flow of population, with applications to the concrete problems of life table construction. The development of these methods centers around the names of Knapp, Zeuner, and Lexis. The author has done an important service in bringing a clear presentation of these methods before American actuaries. In this theory the aggregates of the living and the dead corresponding to assigned birth and age intervals are derived in terms of definite integrals, and the rates of mortality are obtained as the solution of a linear differential equation. The results are made concrete by application to census data.

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