

*Cours de Physique Mathématique de la Faculté des Sciences. Compléments au Tome III. Conciliation du Véritable Déterminisme Mécanique avec l'Existence de la Vie et de la Liberté Morale.* Par J. Boussinesq. Paris, Gauthier-Villars, 1922. xlviii + 217 pp.

Strange title for the third volume of a book on Mathematical Physics! Strange assertion in a strange title! Yet appearing in all simplicity in the closing volume of the new edition of an older publication, now the mature judgment of a distinguished scientist over eighty years of age, the affirmation in fact of a first statement made forty-five years ago! Happy the person who can more confidently state at eighty years of age what he thought was true when he was thirty-five! Member of the Institut, author of many treatises, many memoirs, and minor papers, professor of mathematical physics and of the theory of probabilities—who has a better right to state what he finds, with confidence and serenity?

What is the startling assertion? It is very simple and yet it makes much of scientific philosophy rock preparatory to its fall in ruins. This assertion is that mechanics declares the necessity of a directive principle which is outside mechanics in order to account for the phenomena of the universe. What! Can it be that the most definite of all the branches of physics, itself the most definite and well-understood of the sciences, the branch which even gives its name to that mode of thinking which passes among some scientists as the only straight and narrow path to truth—the mechanistic philosophy—can it be that this discipline itself has turned traitor to the cause, and gone over to the camp of the enemy where freedom smiles, where there are directive principles, entelechies, even perhaps real mind? Is this the outcome of a lifetime study of the laws of mechanics and probabilities? Yet so it is. And indeed he is not without very respectable company in these opening years of a new century, now almost a quarter gone.

The investigation of M. Boussinesq has all the clarity of French mathematics, is easy to read, is not—as he says distinctly—a metaphysical argument, but is the unescapable conclusion of a mathematical consideration of the laws of motion. It is simply this in brief: the laws of motion are stated always in the form of differential equations. These have in many cases, when we come to integrate them, ambiguities in the integrals, due to the fact that the characteristic curves of integration are bifurcated at certain points and also to the fact that these curves often have envelop curves that represent the singular solutions. A particle traveling on a characteristic curve—to take the simplest case—may follow the envelop, for each characteristic curve somewhere is tangent to the enveloping curve. What determines then the path chosen in these cases, since the laws of motion show that the paths are indeterminate, and yet the particles do follow unique paths? The only answer is that at the singular points where a choice of path is possible there must be a directive principle which selects the path. This does not affect the accelerations, therefore does not consist of a force, and hence is not to be accounted for in the energy equations. This becomes the true distinctive character of phenomena due to life, conformably to the extreme physico-chemical instability of living beings,

not imitated by inanimate things. The configuration of the particles of the universe is not altogether explainable by the previous configurations. There were in some situations alternative possibilities, and the laws of nature do not explain why one choice was made rather than the other,—in fact if this could be explained, there would have been no choice and no singularity in the characteristic curves.

The idea that all the phenomena of physics and chemistry can be reduced to modes of motion vanished long ago, for it became evident that the "field" played a very important part in the explanation of phenomena studied by the physicist and chemist, and the field again is described by means of partial differential equations. More complicated singular solutions have to be admitted, as well as singularities of the surfaces of the integrals. This would add force to what M. Boussinesq has to say regarding total differential equations as representing phenomena. It is evident in either case that there may be situations in which the laws of physics—all of which can be stated in mathematical form—fail to account for the following events. This simple fact is enough to invalidate mechanistic hypotheses. For these hypotheses have for their specific aim to do without any other laws than those of physics.

Boussinesq calls attention to a memoir of Poisson's which contains the incipient notion of a directive principle, arising from a consideration of singular solutions. In his *Traité de Mécanique* however he does not mention the matter. Again Duhamel reproduces Poisson's example but does not make much of it. Cournot also considered the matter, but evidently was prejudiced in favor of determinism and sought to find justification for throwing out the singular solutions and the bifurcated paths. Everything considered, there seems to be demanded a dynamics of the directive agencies, which will lead directly from the mechanics of forces to the activities of life, mind, and society. Among the principles for such a science are the principle of simplicity, and that of unity, the principle of variation, and of continuity, the principle of economy or least action. Among the problems he hopes this new science may some day solve is that of the marvelous adaptations of organ to function, of species to environment, usually dismissed with the mere statement that they are necessary for existence.

The volume has several long complementary notes, related to the main memoir. Such topics are the analogy of the mechanism of life with wave-motion, the dissipation of energy and the reversal of purely mechanical movements, the rôle and legitimacy of geometric intuition, the application of the "threshold" of sensation to a possible theory of certain quanta, and a consideration of the impossibility of knowledge of intramolecular phenomena.

This volume is most interesting for those who have philosophic instincts, not only for its positive contribution, but for the fact that it opens the way to a clearer understanding of the freedom that is found to lurk even in mathematical equations. In these days when the pendulum is swinging towards a better interpretation of the facts of life and mind, it is well to comprehend just wherein real determination finds its sphere.

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