Vector-Rechnung. By Dr. Max Lagally. Mathematik und ihre Anwendungen, II. Leipzig, Akademische Verlagsgesellschaft, 1928. xvii+358 pp. $5.50.

Punkt- und Vektor-Rechnung. By Alfred Lotze. Göschens Lehrbücherei. I Gruppe: Reine und angewandte Mathematik. Band 13. Berlin und Leipzig, Walter de Gruyter, 1929. 192 pp. Rm 12.

Vektor-Analysis. By Dr. Siegfried Valentiner. Sammlung Göschen. Berlin und Leipzig, Walter de Gruyter, 1929. Rm 1.50.

The first of these texts is the outcome of lectures covering some years, given by Dr. Lagally in the Technical High Schools of Munich and Dresden, before students who were familiar with physics and mathematics. It is therefore designed to furnish engineers with a rather complete reference book on this subject. The notation so far as possible is that of Gibbs. The book is divided into chapters as follows:

1. Elementary vector algebra, 55 pages; 2. Vectors dependent upon scalar parameters, 62 pages; 3. Theory of fields, 70 pages; 4. Dyads, 49 pages; 5. The most important dyads of mechanics, 37 pages; 6. Transformations, 25 pages; 7. Vectors in Riemann space, 47 pages; 8. Complex numbers, 15 pages.

Each chapter has several sections. The author has compressed into the book a great deal of the current methods labeled "vector." References are not too numerous but sufficient for the purpose. The author has arrived at dyads, or linear vector operators, early in the book, which he considers a desirable thing to do. He tries to give a brief exposition of quaternions and hypernumbers at the end, with ill success. The last chapter could have been omitted.

The claim is made in the preface that the approach is direct and intuitive, but when one finds that this approach is limited to a few diagrams and that the old familiar coordinate system is on the scene rather promptly, and from then on really dominates the development, the vector system sinking as usual to the mere shorthand expression for various coordinate forms, he is not ready to accede to the claim of an intuitive development.

The author recognizes the slow spread of vector methods in current literature, but he does not seem to realize that this slow spread is due to the persistent use of coordinates in at least the thought of those who would use the vector methods. There is not the slightest necessity to mention coordinates or the "Dreibein" (which may be translated *trisceles*, meaning trirectangular axes). Formulas for rotation of the trisceles and other transformations are not needed, and the invariantive character of the expressions is obvious. It seems strange that after these hundred years of ideas of Hamilton and Grassmann anyone should still be clinging to the antiquated methods and forms of coordinates. Anyone familiar with the literature and the discussions about vectors for the last forty years knows that vectors properly developed ("autonomously" in the language of Burali-Forti and Marcolongo) furnish an invariantive mode of thought and writing for geometry and physics. The mere fact that the laws of physics can be so stated shows them as invariants. This simplicity is destroyed as soon as the vectors are made to depend upon axes or coordinates. The only way such dependence should enter is in the representation of variable points, as for instance the vector ρ may depend upon a single parameter when the terminal point P travels on a curve, or on two parameters when it is confined to a surface. Three parameters, like x, y, z, would make it variable in three-dimensional space. For differential purposes these may sometimes be useful, though even under such conditions the free differentiation in any desired direction is much better. For curves the intrinsic vectors, related to the curve, are the only ones that furnish proper directions for differentiation and the same may be said of surfaces. When this is once recognized the clumsy machinery of axes, even when oiled with vector treatment, disappears.

These remarks apply of course to scalar fields and vector fields. It shows strongly in the use of the Hamilton ∇. The present author drags this in with i, j, k attachments, a most common but unnatural method, one which makes it look artificial, and which demands invariant proofs at once. This is of course unnecessary, since the operator may be developed with no reference to coordinate axes, and has been so developed. Further the usual difficulties in numerous formulas appear with the use of the div, rot, grad notation. We cannot enlarge upon this point; it has been discussed before in many papers.

The chapter on transformations is preparatory to that on Riemann space, which follows. It contains a treatment of the "fundamental form" for a three-dimensional space as viewed from the parameter stand-point, but developed as if the ideas were still those of coordinates. It amounts to a vector treatment of curvilinear coordinates. In the following chapter we are led into space of N dimensions, with the notion of curvilinear coordinates still prominent. This is the current method, but it is not necessary to bring in vectors if one is going to do it this way. The criticism that nothing has ever been done by vectors which had not been already done some other way, is perfectly valid when vectors are brought out in this fashion; for the vector method really is nothing more than a shorthand expression of what had already been worked out in longhand.

The text is well printed, and for those who desire knowledge of what is going on in most work related to this subject it is satisfactory. It will not, however, be the starting point for further developments of vector methods.

The second book mentioned above is a rather straightforward treatment of the Grassmann Ausdehnungslehre, and the author considers Grassmann's work as the only existing basis of unification of all vector methods. The attempt is made to produce a vector analysis and a dyadic analysis, but the success is not manifest. The applications are mostly from projective geometry, and for the very good reason that the Grassmann methods are definitely projective. They do not lend themselves to metric treatment readily and it is a mistake to try to get metric results and theorems that way. A small chapter is given on mechanics, and shows at once the limitations of the method. As an introduction to the ideas of Grassmann the book will be fairly successful.

The last book mentioned is merely a reprint of the third edition, the only noticeable change being a table of formulas at the end. It is a brief introduction to the subject as expressed in the current German notation. One small chapter on applications to physics discusses three topics. For a brief glance at vectors it is a good text.

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