DEDEKIND'S WORKS, VOLUME I

Richard Dedekind.   Gesammelte mathematische Werke.   Herausgegeben von
    Robert Fricke, Emmy Noether, Öystein Ore. Erster Band. Braunschweig,
    Friedr. Vieweg & Sohn, 1930. 397 pp.

Richard Dedekind's 85 years (1831–1916) spanned one of the most fruitful
periods in the history of mathematics. His dissertation is dated at Göttingen
in 1852, about two years before the death of Gauss, and he lived to see Ein-
stein's theory of general relativity and gravitation. The peer of his great
contemporaries, Dedekind was probably less widely known than any of them.
Those whose inclinations led them to inspect his more abstract creations recog-
nized the greatness of the work, but by the general public, even among leading
mathematicians, the most characteristic products of Dedekind's genius were
neglected. For it was and still is possible to be a highly competent mathe-
matician without ever having heard of the theory of ideals. This needs
but slight qualification even when account is taken of the infiltration of
modern arithmetical concepts into the theory of algebraic functions and other
branches of mathematics; the ideas of the theory, but scarcely the theory itself,
are achieving a certain measure of popularity for what it may be worth.

Dedekind's material career seems to have been strangely like his master-
piece; he never occupied one of the great chairs in his native country or else-
where. And it would be difficult to find anywhere a more superb illustration
of the usual idiotic logic of history than in the rise of Dedekind's popular
scientific reputation. For one mathematician who knows his way about in the
Dedekind theory of ideals, there are probably hundreds who have been familiar
with the Dedekind cut from their youth up. The thing is notorious; every tyro
is indoctrinated with it in his advanced calculus. And now, according to the
customary irony of time, this cut, for which Dedekind is hundreds-to-one more
famous than for his work in algebraic numbers, seems to be turning into some-
thing like a very bad pun, for it is a schism in the official faiths of analysis
and a prolific source of interminable polemics. Dedekind himself is said to have
hesitated nearly two years before committing it to print.

This first volume of his collected works contains 19 papers in chronological
order from 1852 to 1882. According to the editorial plan, the memoirs shall
occupy the first two volumes, while the two essays on number (as they are
known in America) and the expository French account of the theory of
algebraic numbers (Darboux' Bulletin) are reserved for the third volume. This
will no doubt appeal to most as a wiser arrangement than a strict adherence
to chronological order. The essential parts of the famous supplements on
general arithmetic in the second, third and fourth editions of Dirichlet's
Zahlentheorie are to be reproduced. A life of Dedekind is also promised for the
third volume. The printing is beautifully done on good, restful paper.

The editing of the present volume has been carried through with unusual
care, and with some features of marked interest not often found in collected
mathematical works. At the end of each of the more fundamental papers, the
responsible editor has added short, helpful notes. These are roughly of three
kinds: brief historical indications relating Dedekind's work to that of others;
incisive criticism—in the reputable, relevant sense; references to important

extensions and completions of Dedekind's work by later writers. The arithmetical papers have been edited by Noether and Ore, those on elliptic modular functions by Fricke. Touching the last topic, Fricke calls attention to the much earlier use by Gauss of the "Dreiecksnetz." There does not yet seem to have been discovered a similar anticipation by Gauss of the theory of ideals, either Kummer's or Dedekind's. This is a little surprising, as Gauss could hardly have stopped short—by luck or lack of curiosity—with those fields in which the fundamental theorem of arithmetic happens to hold. Where necessary, the editors have added footnotes, of the same general character as the others.

Whether or not Dedekind's theory of ideals is to be the final one, many believe that it will endure as one of the great landmarks in mathematics. Indeed, the work of Ore has cast a new light on the whole field, showing that by an easier approach the desired end is attainable by strictly finite means. Whatever may be the degree of resemblance between the ultimate theory of algebraic numbers and Dedekind's, the creation of Dedekind will no more be abolished by the new than was Euclid by Riemann or relativity. The fruitfulness of Dedekind's theory is becoming daily more evident.

When a step of the historical importance of Dedekind's theory has been taken, its interest and its influence abide. And it is of some significance to have the history of a such a step exact. In this respect there still seems to be a trifle of haziness, even after the painstaking work of the editors. Without the final theorem of Dedekind concerning certain troublesome divisors, the theory was incomplete; in fact it was not a theory, for without the final theorem it failed to solve completely the problem proposed. It would be of some interest to settle once for all the date of first publication of the complete theory of Zolotareff. The unsatisfactory first draft was the only one known to Dedekind—after he had elaborated his theory; the complete theory of Zolotareff until comparatively recently seems to have escaped notice. Dedekind and Zolotareff worked in total independence of one another. Zolotareff committed suicide so early (a girl, not his theory) that he had no opportunity to exploit his discoveries. That elusive date has a certain human interest as well as historical.

After renewing acquaintance with some of these great papers, and being stirred again by their sublimity of conception, one is slightly relieved to find that Dedekind after all was human. One short note in this volume tells the story. It concerns a trifle about quadratic residues which had attracted Sylvester's gadding attention. From the tone of his remarks about the "Editors of Dirichlet's works"—the *Zahlentheorie*—it would seem that Sylvester was unaware at the time of Dedekind's existence. At any rate he trod on the remiss Editor's toes in truly Olympian fashion. Dedekind, of course, was in the right, until he allowed himself the doubtful luxury of that most human lapse of good judgment, a tart reply in print to a bumptious critic.

Dedekind's style is restrained, at times almost cold. But he put all that was in him into his work, and that all was tremendous. His admirers will hope for a revealing portrait of this great genius in the third volume, which will show him to be something more than a legend and a name.

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