as the continuous ones. But if the discussion of such problems is omitted from the book before us, except for bibliographical references, the reason is obvious and sufficient. It requires a different preparation in analysis from the one which is assumed.

The same remark applies to development of arbitrary functions in terms of various kinds of sets of orthogonal functions. Many facts are given; but many others, as interesting and as important, depend upon a different sort of analysis. The author must have turned the pages of Fatou's famous memoir with regret at the exclusion of what has been a touchstone for modern mathematics.

Enough has been said to show the reader the interesting and accessible character of the book before us. The author is known among mathematicians for his skillful and clear expositions, as well as for his original contributions and scholarship. The exposition here is reinforced by a number of important exercises, some of them given for the sake of review and drill, but many also in order to present interesting facts and illustrations. The work is unusually rich in scholarly bibliographical data.

G. C. EVANS

VOGEL ON EGYPTIAN ARITHMETIC

Die Grundlagen der Aegyptischen Arithmetik in ihrem Zusammenhang mit der 2:n-Tabelle des Papyrus Rhind. By Dr. Kurt Vogel. Munich, Beckstein, 1929. 211 pp.

This book, an inaugural dissertation of the University of Munich, is an important contribution to the history of mathematics. It is of especial interest to American scholars because its appearance is almost simultaneous with that of Chancellor Chace's monumental edition of the Rhind Papyrus, published by the Mathematical Association of America.*

Dr. Vogel's book is divided into two main divisions, first, a general discussion of ancient Egyptian arithmetic (pp. 5–53) and second, the $2:n$ table of the Rhind Papyrus (pp. 53–181). These parts are preceded by a four-page introduction, discussing the place of the Rhind Papyrus in the history of mathematics; and are followed by a resumê of the chief results obtained, pp. 181–195. There are good indexes and a bibliography of 106 titles.

The first part gives a very clear and satisfactory account of Egyptian arithmetic with particular attention to the use of fractions, including an explanation of the technical expressions used in the Rhind Papyrus and also a brief description of ancient Egyptian weights and measures. The methods of the Egyptian calculator are illustrated by frequent examples from the Rhind Papyrus. It is made clear, among other things, that we are justified in crediting him with considerable skill in mental arithmetic, as well as with a thorough grasp of the fundamental processes, including division by a fraction. Finally, Vogel is able to give a definite answer to the question, Did the ancient Egyp-

* Reviewed by D. E. Smith in this Bulletin, vol. 36 (1930), pp. 166–170.

tians have a concept of the *general* fraction? (It is well known that they had no way of *writing* fractions with numerators greater than unity.) The answer, in the affirmative, is well supported by all the material so well marshalled in this book; and moreover the opinion of certain other scholars* who had previously been skeptical on the point can now be quoted as in agreement with the conclusion, which Vogel states thus (p. 185): "Es wird also jetzt von allen Seiten anerkannt, dass der Aegypter den klaren Begriff des allgemeinen Bruches (in dem nicht-komplexen Sinn) gehabt hat."

The second part is even more important, and contains the author's chief original contributions to the subject. He divides the work into 4 chapters, (1) the theory of partial unit fractions for $2/n$, (2) a study of the actual results stated in the Rhind Papyrus, (3) the two exceptional cases, and (4) the probable development of the table in the Rhind Papyrus. The summary and conclusion occupy the last 15 pages.

In the first chapter (pp. 61–103) the separation of $2/n$ into partial unit fractions is considered from the standpoint of the formula

$$\frac{2}{n} = \frac{1}{x} + \left(\frac{1}{K_1 n} + \frac{1}{K_2 n} + \cdots + \frac{1}{K_\nu n}\right)$$

where $1/x$ is called the "principal fraction" and the parenthesis the "remainder." Since the Rhind Papyrus contains no case where $2/n$ is broken up into more than four unit fractions, we have to consider for ν only the values 1, 2, and 3. Doing this, Vogel is able to derive from the formula all the results of the table in the Papyrus (along with a great many others, naturally) with the exception of 2/35, 2/91, and 2/101, the last of which may be disregarded as trivial, being given as

$$\frac{2}{101} = \frac{1}{101} + \frac{1}{202} + \frac{1}{303} + \frac{1}{606},$$

which simply uses the representation of 2 as

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6},$$

but which is not in accordance with the Egyptian's otherwise universal rule to take his "principal fraction" greater than $1/n$. As his next step, Vogel modifies his procedure to develop the theoretically possible representations on the basis of the "principal fractions" actually used by the Egyptian in the Rhind Papyrus. For a separation of $2/n$ into only *two* unit fractions, of course the choice of $1/x$ determines uniquely the other, or "remainder" fraction, thus.

$$\frac{2}{n} = \frac{1}{x} + \frac{2x - n}{xn},$$

where $2x-n$ must accordingly equal either 1 or a factor of n. This happens in 28 cases out of the 50 in the table. Of the others, it is sufficient to say here that the separation into three or four unit-fractions leads to divisibility considera-

* Particularly, Peet and Neugebauer.

tions of some complication, so that, as Vogel does not fail to mention, it is quite impossible that we have here the actual method by which the Egyptian computed his table. But the discussion is none the less of interest. Two cases indeed ($n=35$ and $n=91$) do not come under the formula just given at all, and so are made the subject of a separate examination.

In the second chapter (pp. 103–157) the actual table of the Rhind Papyrus is carefully analyzed and the results classified in 5 groups (omitting the two exceptional cases just mentioned). Along with this (pp. 115–128) is a copy of the table itself, with valuable critical notes both on the Papyrus and on Peet's edition.* The main question that now remains to be answered is whether there is any discernible logical principle, any system, by which the Egyptian obtained, first his "principal fraction," and then the "remainder", in cases where this was broken up into two or more parts. Vogel accordingly gives us first, in concise fashion, the conclusions as to this fundamental question that have been reached by all the important previous writers, including Eisenlohr, Cantor, Favaro, Griffith, Hultsch, Vetter, Peet, Gunn, Neugebauer, Bobynin, and Gillain.†

Before giving his own definitive answer to the question, Vogel gives in the third chapter (pp. 157–173) an explanation of the "exceptional cases" $n=35$ and $n=91$, for which the partial fractions given are

$$\frac{2}{35}=\frac{1}{30}+\frac{1}{42}, \qquad \frac{2}{91}=\frac{1}{70}+\frac{1}{130}.$$

It is shown that the method can reasonably be used not only in these two, but in all the other more or less doubtful cases.

In the fourth chapter (pp. 173–181) Vogel sums up his argument with the final conclusion, which seems thoroughly justified by the preceding study, that the choice of the "principal fraction" was made more or less arbitrarily from a number of empirically observed possibilities, but that when this had been done a definite method of procedure was always used, which led to a unique result for the "remainder." He then discusses the possible evolution of the table, its purpose, and its influence upon the arithmetic of the Greeks, Arabs, and medieval Europeans.

Altogether this book is an excellent piece of work, done in a scholarly and interesting fashion.

R. B. McCLENON

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* Reviewed by D. E. Smith in this Bulletin, vol. 30 (1924), pp. 557–559.

† References in detail can be found in the *Bibliography of Egyptian Mathematics* by R. C. Archibald, in Chace's edition of the Rhind Papyrus, vol. I, pp. 127–191.