SHORTER NOTICES

Identität und Wirklichkeit. By Émile Meyerson. Translated from the third French edition by Kurt Grelling; with Introduction and Notes by Leon Lichtenstein. Leipzig, Akademische Verlagsgesellschaft, 1930. 40+534 pp.

The philosophy of Émile Meyerson has been expounded during the last twenty years, in four large volumes, the first of which has now appeared in its third edition; it is this edition which has formed the basis for the German translation that is before us for review. It contains besides the author's preface and three appendices (on *Leibniz, Newton, and action at a distance*; on *The followers of Copernicus and the principle of inertia*; and on *Herakleitos* πάντα ρει), an introduction to the philosophy of Meyerson by Lichtenstein. This introduction makes clear that the work of Meyerson is of especial interest and significance for mathematicians as well as for those concerned with the sciences. A detailed examination of the bearing of Meyerson's work upon the philosophical foundations of mathematics would doubtless be of great value. It must suffice here to say that his fundamental doctrine refers to the conflict between the nature of the human mind and that of nature outside man, and that his method consists in an historical survey of the manner in which scientific conclusions have been arrived at.

ARNOLD DRESDEN

Das Unendliche in der Mathematik und seine Ausschaltung. By Felix Kaufmann. Leipzig and Vienna, Franz Deuticke, 1930. x+203 pp.

This book advocates nothing less than the elimination of the infinite from mathematics. At first this seems a bit startling (though the idea itself is hardly original), but it soon turns out that the author's viewpoint is not very far from that of Brouwer and the intuitionists, on whom he leans quite heavily.

The author's major interest seems to be in philosophy, but one learns from the preface that he has been interested in the foundations of mathematics for thirteen years, and for six years has taken part in the discussions of the foundations undertaken by a circle of philosophers and mathematicians of Vienna under the leadership of M. Schick and H. Hahn. He acknowledges the cooperation of A. Becker, A. Fraenkel, and K. Menger.

The book itself, apart from its thesis, can be recommended as a pleasant introduction to the foundations of mathematics in the present controversial state of that subject. It contains a good account of the positions of Brouwer, Hilbert, Russell, and Fraenkel. Especially welcome is the clear presentation of the intuitionist case. The chapter headings are: Einleitung; I. Grundtatsachen der Erkenntnis; II. Symbolik and Axiomatik; III. Natürliche Zahl und Menge; IV. Negative Zahlen, Brüche, und irrationale Zahlen; V. Die Mengenlehre; VI. Das Problem der durchgängigen Entscheidbarkeit arithmetischen Fragen; and VII. Die Antinomien.

If we accept Brouwer's view, the only sets which exist are those which are countable and have been effectively well-ordered. The author remarks, and this is his principal point, that we can as well go the whole way and admit the existence only of finite sets. Any statement about a countable, effectively well-ordered set can by a circumlocution be translated into a statement about the rule by which the elements follow each other in the well-ordering, which rule is something finite and definite. For example, we are shown how it is possible in his scheme to prove that every bounded monotonic sequence of irrational numbers has a limit. Of course both the sequence and the irrational numbers composing it must be given effectively, since for the intuitionists only those irrational numbers exist which have been effectively defined.

The author begins the introduction by remarking that just as the nineteenth century witnessed the banishing of the infinitely small from analysis through the work of Weierstrass and others, so he will show that we can (and must) dispense with the actual infinite of Cantor. This analogy is not perfect, however, as there exist quite respectable number systems and geometries with absolute infinitesimals, even if they have no place in the ordinary function theory.

The first two chapters contain the philosophical ideas of Husserl and Brouwer which have led the author to his conclusions. These may be of less interest to the mathematician than the succeeding chapters where these ideas are applied to the outstanding problems of the foundations of mathematics. The third and fourth chapters deal with the foundations of the real number system. The fifth chapter discusses general set theory, ordinal numbers, and the various attempts to put the theory of sets on an axiomatic basis. The sixth chapter raises the question of the "categoricalness" of the system of positive integers. Can every statement about the positive integers be either proved or disproved? The conclusion is reached that at least it can never be proved that the answer to this question itself is "No." The last chapter concerns the paradoxes of the infinite. It is not suprising that these should cease from troubling in a system from which infinite sets are excluded.

It seems to me that the author, besides producing an interesting book, has made a good case for the contention that if we accept Brouwerism, we can get along theoretically without the notion of an infinite set, whether or not that notion is meaningless, as the author maintains. However, I do not believe that the views of Brouwer will ever find general acceptance among mathematicians. As this is not the place for an elaborate discussion of the questions raised by the intuitionists, I should merely like to add the following minor point to the prevailing confusion. If the continuum hypothesis is true, it is conceivable that someone may someday discover an effective way of well-ordering the real number continuum so that every number has only a countable number of predecessors. If this were done it would be practically a refutation of Brouwerism. It might seem, then, that either the intuitionists must prove that it cannot be done, or must proceed with a sort of sword of Damocles hanging over their heads. But it would be a violation of intuitionist principles even to attempt such a proof, to so much as mention the real number continuum. The intuitionist contentions are not themselves based on proof. On the other hand they require that mathematicians be very fussy about their mathematical proofs and avoid the use of reductio ad absurdum arguments and the axiom of choice.

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