

tion of these conceptions. It reveals that bodies of propositions have formal properties and that they can be handled as a whole. Moreover, these formal properties of bodies of propositions prescribe rules which are quite independent of any specific subject matter whatever. In technical language, the properties of the system remain the same if variables are substituted for constants, thereby turning each proposition into a propositional function, and the system or doctrine as a whole into a system or doctrinal function.

Unconsciously science has been taking advantage of this fact. Thus much of scientific procedure consists in establishing identifications between the variables of some purely mathematical system and the physical constants of some empirical subject-matter. Thus instead of working by observation from isolated facts to general principles, or from a hypothesis, put up more or less arbitrarily, to the verification of its deductive consequences, science confronts the actual system of an entire empirical subject-matter with the possible doctrine functions of pure mathematics. Einstein's identification, in the general theory of relativity, of the potentials of the gravitational field with the  $g_{ik}$ 's of a tensor form in pure mathematics is a case in point.

This is but one example of the many ways indicated in Carmichael's book in which the modern logic of doctrinal or system functions forces us to modify our conceptions of important things. It is easily the most readable treatment of this aspect of modern logic that has been written.

F. S. C. NORTHROP

*Lezioni di Geometria Analitica e Proiettiva*; parte seconda. By Annibale Comesatti. Padova, Casa editrice A. Milani, 1931. xii+440pp.+57 figures. Price Lire 60.00.

This second volume of the comprehensive treatise on the basic concepts of analytic geometry is concerned entirely with projective geometry of one, two, and three dimensions, treated analytically and restricted to entities generated by linear forms: points, lines and planes.

While the results obtained in the preceding volume (reviewed in this Bulletin, vol. 36, p. 474) are occasionally cited, it would be possible to read the present one without any knowledge of it further than the general concepts of coordinates and of the elements of projective geometry, treated synthetically.

The general plan of this second volume is much more closely related to our American scheme of instruction than was that of the first. It does not treat a wide range of subjects, but develops the one concept of projective transformations systematically and exhaustively. Following each chapter is a generous list of exercises, and the reader is urged to supplement the algebraic discussion with a free use of synthetic confirmation.

After binary collineations in a purely projective field have been introduced, they are applied to various metrical cases, including pencils of circles and groups of motions. In the ternary field the ordinary problems of projection and section, homology, involutions are first presented; then follow the types of plane collineations, cyclic collineations, and numerous metrical particularizations. Duality and polarity are given an entire chapter, which includes the duality between a bundle and a plane field.

In space of three dimensions more details are given in the development of homogeneous point coordinates; then follow the canonical types of collineations

and the geometric meaning of each. Duality includes the null system, with linear systems of linear line complexes, and the tetrahedral complex.

The last chapter of the book is devoted to a discussion of the foundations of projective geometry from the axiomatic standpoint; it includes postulation, numerical spaces, ordinal relations, harmonic groups, equivalence and the postulate of Archimedes.

This is followed by a development of projective coordinates of imaginary elements, with particular emphasis on the invariance of cross ratios.

The book is excellently printed with black type on thin opaque paper, is practically free from typographical errors, and is provided with a subject index and a list of authors. It provides an adequate and dependable preparation for work in algebraic geometry.

VIRGIL SNYDER

*Georg Cantor.* By A. Fraenkel. Leipzig, B. G. Teubner, 1930. 78 pp.

In view of present day interest in the foundations of mathematics, this pamphlet is a timely publication. The major portion is naturally devoted to a sketch of Cantor's life and personality, and a full discussion of his development as a mathematician. As might be expected, these are inextricably mingled. Although the *Mengenlehre* holds the center of the stage, some of Cantor's other activities are of no less interest to the reader,—for example, his concern with the Bacon-Shakespeare controversy.

The latter part of the pamphlet is concerned with an outline of Cantor's publications and ends with a bibliography. It is of interest to learn of his contributions to the theory of numbers and the theory of trigonometric series, and instructive to see how these led to the theory of aggregates. It is in precisely such respects that a work of this kind is of value, providing the student with a background that gives a richer knowledge of the subject.

W. A. WILSON

*Cours de Mécanique.* By Paul Painlevé and Charles Platrier. Paris, Gauthier-Villars, 1929. iv+644 pp.

This book is a printed version of lectures given by C. Platrier (substituting for P. Painlevé) to second-year students at the École Polytechnique. The main topics discussed are Rigid Dynamics, Hydrodynamics, Elasticity, Aerodynamics, Theory of Relativity. The point of view is quite modern and the treatment is very satisfactory. The discussion of the gyroscope is about the best we have seen in a general treatise as is also that on wave-propagation. The discussion of tensor analysis is adequate for a student who is not desirous of being a specialist in this subject. The only criticism that one could fairly make of the work is its very academic character; a book containing several chapters on aerodynamics which does not mention Prandtl's name cannot appeal strongly to the practical man of affairs. Nevertheless the book is a valuable addition to Appell's renowned treatise on Mechanics, which book it (together with Painlevé's *Cours de Mécanique*, Tome I) replaces to a certain extent.

F. D. MURNAGHAN