Vector Analysis. By H. B. Phillips. New York, John Wiley and Sons, 1933. viii+236 pp.

The following quotation from the preface will give some idea of the contents: "In broad outline the book consists of two parts. The first five chapters cover the fundamental operations and the more general properties of scalar and vector fields. The remaining chapters contain the detailed analysis of fields, the properties of potentials, and linear vector functions. In an elementary course the work might be restricted mainly to the first five chapters together with selected topics from the others." The first part includes, in addition to chapters on the algebra and calculus of vectors, a discussion of line, surface, and volume integrals, Stokes's theorem, the divergence theorem, and generalized coordinates. The last five chapters contain applications to electricity and hydrodynamics, an introduction to potential theory, and a chapter on dyadics. Although the material covered can hardly be classified as easy for an undergraduate, the author has succeeded in making his presentation as elementary as the nature of the subject matter allows. There are 235 problems. We believe the book contains excellent material, well presented, for a semester's work following advanced calculus.

The following comment is not to be taken in any way as a criticism of the book under review. It is our opinion that the usefulness of the Gibbs vector notation is much overrated. For most purposes the representation of a vector by a typical component, using the symmetric subscript notation and the double index summation convention, is far more efficient. For an exposition of this notation the reader is referred to the recent book, *Cartesian Tensors*, by Jeffreys, or to *Vector Analysis and Relativity*, by Murnaghan.

C. A. SHOOK

Differentialgeometrie. By L. Bieberbach. Teubner's Mathematische Leitfaden, Volume 31. Berlin, Teubner, 1932. vi+140 pp.

I quote from the preface: The purpose of this "Leitfaden" is to give an introduction to the differential geometry of real curves and surfaces of the euclidean plane and euclidean space. The endeavor is made throughout to do this with the least possible prerequisite knowledge. In general only the elements of the differential and integral calculus and of analytic geometry are necessary, the last indeed in vector form . . . . "Ich hoffe weiter, dass man in meinem Buche kein saloppen Gedankengänge finden wird, keine unsauberen Schlüsse von der Art wie sie auch die moderne Literatur über Differentialgeometrie leider so oft noch beherbergt—als Überreste aus der Plüschmöbelziet und als Verstandesschoner."

The book is an extraordinarily good, rigorous, and short presentation of the principal elementary results of differential geometry with the inclusion of a good deal of interesting matter not usually to be found in a first book on the subject. There are only 132 pages of text, which are divided into three chapters: 1. *Curves in the Euclidean Plane* (pp. 1–29); 2. *Curves in Euclidean Space* (pp. 30–46); 3. *Surfaces in Euclidean Space* (pp. 47–132). The last ten pages of the third chapter give an introduction to the tensor calculus.

The presentation of the subject is clear and careful; much more attention is paid to rigor than is customary in books on Differential Geometry. The treatment is extremely compact, a fact mostly due doubtless to the use of vector methods. The author has a light touch and frequently brings a smile to the face of the reader by his sarcastic comments on other (unnamed) writers. I think the book a very good one for anyone familiar with the subject, but that it should serve a beginner as a "leading string" seems to me not possible; that a student with the slight mathematical knowledge supposed to be necessary for the study of the book should be able to learn the subject from it or to acquire an interest in it does not seem likely. The discussion is somewhat uneven, often rather difficult; again, when the author remembers that he is writing a "Leitfaden," of extreme simplicity. I question whether the use of the vector method is the best way to begin the study of differential geometry, whether the use of machinery almost inevitably strange and difficult for a student of modest equipment will not distract his mind from the great geometrical interest of the subject. Without doubt the method is generally short and elegant, but there are occasions where to me it seems forced and clumsy. Moreover I am not convinced that the vector method furnishes a very good introduction to the tensor calculus; I am doubtful whether an introduction to the tensor calculus should appear in a "Leitfaden" in differential geometry, for I am not yet persuaded that the chief purpose of classical differential geometry is to serve as an introduction to the geometry of n dimensions.

Certainly there is little in the book with which one may find fault, but I do deprecate the writing of the Christoffel symbols in their historic forms when the summation convention is to be used. This convention is common and is very useful; it is introduced in the usual way on page 47 of the book. The two Christoffel symbols are introduced on pages 61 and 62,—the symbol of the second kind, it may be noted in connection with the second footnote of page 25, in a highly artificial way. The symbols are used continually with the summation convention throughout the remainder of the book. As the symbols are written the upper and lower indices are in total confusion. I think that Bieberbach should follow L. P. Eisenhart in his change in the writing of the symbols, as given in his *Riemannian Geometry* on page 17.

On pages 92–95 there is a long proof of the very interesting Gauss-Bonnet integral formula, generally omitted in elementary texts, with applications in the following pages. To this proof is appended in a footnote heavy sarcasm to the effect: "I do not know whether one may conclude that those authors have a proof which is so short that it may be left to the readers of a textbook." I hasten to state that I do not question the essential correctness of Bieberbach's proof, but he is, in his proof, chiefly concerned with the evaluation of an integral, $\int d\omega$, where ω is the angle measured from the vector tangent to a parametric curve to the vector tangent to the curve along which the integral is taken; the integral is found by the integration of $d\omega/ds$ with respect to s, the arc. The result is immediately applied to a vertex where ω changes but ds is zero. In the same proof on page 94 is the statement: "the total angle turned by the tangent as it (the point of contact) describes the curve, is, from considerations of continuity already used, equal to the total angle turned by a chord joining points sufficiently near." I can not help thinking of plush furniture. But I think too that this is a very excellent book.

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