SHORTER NOTICES

L'Univers en Expansion. By Henri Mineur. (Actualités Scientifiques et Industrielles, No. 63.) Paris, Hermann, 1933. 41 pp.

This monograph is a good collection of the results obtained by the Lemaître-Eddington-de Sitter line of development of the theory of the "expanding universe." Like those whose work he reports, he is apparently unaware of the existence of Friedmann's 1924 article in the Zeitschrift für Physik on open spaces, and is unfamiliar with the reviewer's 1929 treatment of the subject in the Proceedings of the National Academy of Sciences.

H. P. ROBERTSON

La Théorie de la Relativité. By A. Einstein. Translated into French by Maurice Solovine. Paris, Hermann, 1933. 109 pp. +1 plate.

This pamphlet is a translation into French of three articles, two of which were published in German and the third of which was written by Einstein expressly for this collection. The first two, occupying the first 98 pages, are Einstein's fundamental 1916 article in the Annalen der Physik and the 1931 Einstein-Mayer Sitzungsberichte paper on the ("5-vector") unified field theory. The remaining article is an exposition of the general Friedmann universe (with the usual erroneous attribution of the open-space cases to Heckmann), with special emphasis on the case in which the cosmological constant and the curvature of space are both taken as zero.

H. P. ROBERTSON

Gleichgewichtfiguren Rotierenden Flüssigkeiten. By Leon Lichtenstein. Berlin, Springer, 1933. viii+175 pp.

This important work of the late Professor Lichtenstein was published not long before his death. In it he has given a somewhat brief historical account of work done on the problem by his predecessors, and a fuller account of the contributions made by himself and his students during the last few years. Since he presupposes on the part of his readers a familiarity with the theory of the newtonian potential function and of integral equations, together with a good grasp of other branches of analysis, geometry, and celestial mechanics, the book is not easy to read, but the results are so significant that it should be carefully studied by everyone who is seriously interested in the mathematical treatment of the problem of figures of equilibrium of rotating fluid bodies.

Suppose a body of fluid occupying a region T bounded by a surface S is rotating about an axis with an angular velocity ω. Under what circumstances will it be in relative equilibrium assuming that its parts attract each other according to the newtonian law? The importance of this problem from the point of view of the theoretical shape of heavenly bodies and of their evolution is obvious. It has been studied by a long line of famous mathematicians, including conspicuously Newton, Maclaurin, Jacobi, Clairaut, Tschebyscheff, Poincaré, G. H. Darwin, Liapounoff, Wavre, and the author of this book. If V is the newtonian potential of the body at a point P, and if R is the distance of P from the axis of rotation, then for equilibrium the function $F=V+\omega^2R^2/2$ must be constant on the surface S. The potential V is expressed as a triple integral over the region bounded by S of a function of P and the variables of integration. Under such circumstances to determine S such that the resulting function F will be constant on S is clearly a problem of transcendental difficulty. Fortunately the function V is tractable for a homogeneous ellipsoid, and Maclaurin and Jacobi were able to show that if ω is small enough there are certain ellipsoids which are figures of equilibrium when the fluid is homogeneous. The relative lengths of the axes of these ellipsoids are functions of the angular velocity, there being two essentially different ellipsoids for each small angular velocity. As ω is varied these ellipsoids form two linear series of figures of equilibrium, the ellipsoids of the two series coinciding when ω has attained a certain size. This particular value of ω is thus a point of bifurcation, and is especially important since it suggests the possible existence of other linear series (not ellipsoids) which may have points of bifurcation with the ellipsoidal series. In 1884 Tschebyscheff, and in 1885 Poincaré (probably independently) made use of this idea to find new figures of equilibrium in the neighborhood of the ellipsoidal ones. Assuming the existence of solutions of a certain form, they proceeded to find approximations to the solutions, but in these celebrated papers the actual existence of the solutions was not established. About twenty years later Liapounoff, in a fine series of papers, completed the existence proofs and made very valuable extensions of the discussion of the general problem. His method depended in effect on the solution of a non-linear integral equation of a type discussed by E. Schmidt in 1908, but was made without the aid of the theory of integral equations.

Lichtenstein has made the solution of the problem depend on that of an integro-differential equation, and his method has the advantage of applying not only to solutions in the vicinity of the ellipsoidal solutions but also to those in the vicinity of any series of solutions. He is able to make extensive use of the theory of integral equations, which greatly simplifies the work as compared with that of Liapounoff. In the volume before us, he gives details of some of his existence proofs both for figures previously discussed and for certain new figures of equilibrium. His discussion includes not only the case of a homogeneous rotating fluid body, but also a considerable number of other interesting cases. For example, he discusses the case of a body made up of a finite number of layers each of which is homogeneous, the case of a rigid body covered with a homogeneous fluid, the case of a body with a variable density which varies monotonically from outside to the center, the case of figures which are approximations to anchor rings, and the case of two separate bodies. In this book he does not take up the question of stability of the figures of equilibrium, but references are given to memoirs on this very important matter. Nor does he discuss the exceedingly interesting question of the evolution of the form of a rotating fluid body whose density, mass, and rate of rotation change under conditions which might exist in the evolution of a celestial body; I regret that he did not find it possible to include at least historical references to this matter. A large number of problems closely related to those discussed in Lichtenstein's book await solution.

E. J. MOULTON