systems of coordinates, methods of mapping, linear transformations, and an exhaustive discussion of the linear complex, pencils of complexes and of reguli.

The remaining sections are devoted respectively to algebraic complexes, congruences, and ruled surfaces. The quadratic complex and systems of non-singular quadratic complexes are discussed in such detail as to supply a helpful addition to existing literature in the way of a hand-book. Congruences are nearly as fully treated, especially those contained in a quadratic complex. Applications to the Kummer surface are even better coordinated than in the existing textbooks.

The summary of algebraic ruled surfaces, comprising over a hundred citations, is exceptionally well done. Since it was written, the only important additions are the book of Edge and the essay of Wiman in the Acta Mathematica.

The report was published in the most trying post-war period. This explains the omission of three chapters: differential line geometry, geometry of the sphere, and the use of other space elements, each of which could contribute materially to the usefulness of the report.

VIRGIL SNYDER

Algebraische Raumkurven und abwickelbare Flächen. By K. Rohn and L. Berzolari. Band III 2 B, Heft 9. July, 1926. Pages 1229–1436.

This volume covers approximately a century of research on space curves and developable surfaces. The author is well fitted for the task, having been familiar with and a contributor to the field for more than a third of that time. There is an abundance of material and its arrangement shows very careful planning, for it reads more like a textbook than the source book it really is.

In some places the material is covered in full, in others it is barely sketched. However, in either case the reader is liberally supplied with references. The references given are frequently really footnotes as they contain additional and valuable information upon the subject.

The first two-thirds of the book deals principally with general properties of space curves and their developable surfaces. It begins with some definitions and representations of a space curve, and includes many of the fundamental properties resulting from these definitions. Singular points are introduced early, followed by the Cayley and other formulas. The geometry on a curve and the classification of curves are discussed at some length.

The latter third of the book deals primarily with particular curves and surfaces. Rational curves of order four, five, six, and seven, and irrational curves of order four, five, and six are discussed. Some other special curves as well as special systems of curves are also discussed.

AMOS BLACK

Spezielle Algebraische Flächen. By W. Fr. Meyer. a. *Flächen dritter Ordnung.* Band III 2, Heft 10. 1928. Pages 1437–1531; b. *Flächen vierter und höherer Ordnung,* Heft 11. 1931. Pages 1533–1779.

The discussion of cubic surfaces is divided into two parts. The first includes the older historic treatment, mapping, and the development of properties; the second part reviews the prize papers of Cremona and Sturm, and outlines the Segre projection, Juel's topologic cubic surface, the theory of binary invariants applied to space cubic curves, and the theory of groups.

The surfaces of the fourth and higher orders can not be so easily classified, and the author considers his material in thirteen sections, confining his attention almost entirely to special types of quartic surfaces. The reader is given an introductory survey of curves and invariants on the quartic surface and of Kummer's essay on the F_4 with a nodal conic, followed by a discussion of the F_4 with a double conic developed according to Clebsch's method of mapping the surface on a plane and including the cyclides; of the F_4 with a double line, the $F_5, F_6, \cdots$, with singular curves, and rational surfaces; and of the F_4 with a triple point, the Steiner surface, and the F_4 with a three-fold line. The F_4 with less than sixteen double points comes next, followed by the Weddle and Kummer surfaces, with the surface of Cayley's tetrahedroid, and its metrical interpretation as Fresnel's wave surface. The line geometry of the Kummer surface as well as its representation by hyperelliptic functions of two variables is included. The treatise closes with a consideration of ruled surfaces and of the metric F_4, emphasizing those derived from quadrics.

In the first volume the theory of the twenty-seven lines on the cubic surface is treated at some length, due emphasis being given to the surfaces with multiple points. Various methods of classification and of generation are based upon this theory. The several processes of mapping are presented in such detail that the reader can get a complete and accurate grasp of the subject. A particularly valuable feature is the careful analysis of existence proofs, postulation, and porisms. A few omissions of recent papers have been noted but on the whole the literature has been well covered and the topics skillfully arranged.

There is more overlapping of material in the various sections of the second volume, due perhaps to the fact that surfaces of above the third order do not lend themselves to classification under a small group of distinct headings. Practically all known theorems of a general nature are negative, for example, the statement that a general surface of order greater than three contains no curves simpler than the plane sections.

The topics treated are generally explained fully in the text, affording a comprehensive survey. In the compilation of such an encyclopaedic character as here attempted it is obviously impossible to give complete references, but it is regrettable that so many papers have not received recognition. Several additions have been given in the appendix, but the list remains very incomplete. Godeaux and Sharpe have each been referred to only once, and Snyder's numerous articles including his classification of Dupin cyclides and his list of ruled surfaces of order six (which is complete according to Wiman) have been omitted.

Not infrequent typographical errors have been noted, the majority referring to dates of the papers cited. An example of confusion thus caused is seen on page 1543 in note (4), where the Valentiner reference should be page 22 instead of page 223; the de Jonquières reference (Comptes Rendus, vol. 106 (1888), pp. 209, 526, 907) should not be divided into two separate references (vol. 106 (1887), pp. 526, 907, and vol. 107 (1888), p. 209); and the reference to P. W. White (Cambridge Philosophical Society Proceedings, vol. 21 (1920), p. 116) should be to F. P. White (vol. 21 (1922), p. 216). This last reference has also been given incorrectly in a different form on page 1518. On both pages 1501 and 1611 the paper attributed to G. Veronese (Mathematische Annalen, vol. 24 (1884), p. 313) is an article by C. Segre; the Veronese reference should be Mathematische Annalen, vol. 19 (1881), p. 161. Other types of misprints occur, as on page 1465, line 19 up, C_4 should be C_3; Sekanten should be Bisekanten; on page 1495, line 19 up, Doppel should be Torsal.

With the completion of these two volumes the study of algebraic surfaces will be greatly facilitated. Notwithstanding the omissions, they make a notable addition to the essential literature of the subject. A laborious work has been ably completed.

EVELYN CARROLL-RUSK

Algebraische Transformationen und Korrespondenzen. By L. Berzolari. Band III 2 B, Heft 12. 1933. Pages 1781–2219.

This is the twelfth and last number of the volume on algebraic geometry, and it brings the standard of excellence of the Encyklopädie to a new high level. Berzolari has done an admirable piece of work in this well written, carefully organized discussion of algebraic correspondences and transformations. He has woven the researches of some 960 authors into a harmoniously unified treatment of the subject which will be of immense assistance to algebraic geometers. This book overlaps to some extent Bulletins 63 and 96 of the National Research Council, but to a greater degree it correlates and supplements these bulletins.

The reviewer finds no essential omissions and no errors of fact or typography. The book is divided into eight quite unequal divisions which might be called chapters, and a brief summary of the topics considered will give some idea of the thoroughness of the author's study.

I. *Einleitende Definitionen und Eigenschaften.* This chapter is concerned with such topics as the definitions of algebraic correspondences between algebraic manifolds, reducible and irreducible correspondences, and branch points. Invariant relationships, such as the formulas of Zeuthen and Noether, are discussed.

II. *Algebraische Korrespondenzen und Korrespondenzprinzipien in linearen und nichtlinearen Gebieten.* Algebraic correspondences between two rational curves, (2, 2) correspondences, multi-linear correspondences, and various theorems on correspondences are developed in this chapter.

III. *Algebraische Korrespondenzen und Korrespondenzprinzipien für algebraische Kurven beliebigen Geschlechts.* This is the longest chapter in the book and covers some 125 pages. Among the many topics which are taken up are the Cayley-Brill theorem, systems of several correspondences between the points of a curve, the correspondence theory of A. Hurwitz, Severi's geometrical treatment of the general correspondence theory, and the valence of a correspondence according to Burkhardt and Zeuthen. There is also a discussion of algebraic correspondences between algebraic curves from the point of view of analysis situs, and items such as Zeuthen's rule for the multiplicity of a coincidence point in an algebraic correspondence, grade and genus of a correspondence, symmetric and semisymmetric correspondences. The transcendental theory is brought in by a discussion of the work of C. Rosati and of G. Scorza on Abelian