SHORTER NOTICES

L'algèbre abstraite. By Oystein Ore. (Actualités Scientifiques et Industrielles, No. 362.) Paris, Hermann, 1936. 52 pp.

Ore's pamphlet is the sixth in the series: Exposés d'analyse générale, edited by M. Fréchet. It gives an excellent introduction to so-called abstract algebra. The fundamental concepts are defined and their significance explained. The most important theorems are stated and the direction of modern development is indicated. Proofs are not given but reference is made to a large number of books and papers.

The subjects treated are: the theory of fields under their different aspects, commutative rings and ideals, non-commutative rings, systems of hypercomplex numbers and their representations, some parts of the theory of groups, and finally structures, which last theory has been developed by Professor Ore's own investigations.

The book is clearly written and should be a great help to anyone desiring to form some general idea of this part of mathematics.

It is certainly a difficult task to present so many theories on so few pages. These difficulties Professor Ore has overcome in an admirable manner.

RICHARD BRAUER

Wahrscheinlichkeit Statistik und Wahrheit. By R. von Mises. Einführung in die neue Wahrscheinlichkeitslehre und ihre Anwendung. 2d revised edition. Edited by P. Frank and M. Schlick. (Schriften zur Wissenschaftlichen Weltauffassung, vol. 3.) Vienna, Springer, 1936. 8+282 pp.

The approach of von Mises to probability by means of "collectives" is now so well known that a detailed review of the second edition of *Wahrscheinlichkeit Statistik und Wahrheit* is hardly necessary. He defines a collective as a *sequence of observations*—and there is the nub of the difficulty. An observation is not a concept usually met in pure mathematics, and requires careful explanation. There are severe requirements laid on a sequence of observations which is to be called a collective, and it has been pointed out by many writers that these requirements are impossible of fulfillment if by a *sequence of observations* is meant a fully defined sequence of symbols, say numbers, representing the observations.* On the other hand, von Mises is apparently unwilling to go to the other extreme and state explicitly that by a *sequence of observations* he means a sequence of actual observations of an experiment which is being repeated indefinitely, but whose infinitely many results can never be obtained all together. (The rules of the *Regellosigkeit* axiom would then be decided on before the experiment started, and the axiom would become a prediction of future experimental results—a description of physical occurrences rather than a mathematical axiom.)

* More accurately: the *Regellosigkeit* axiom has no precise significance if this is to be the meaning of a *sequence of observations.*

Von Mises has attempted to take an intermediate position between these two points of view. His own point of view is not clearly defined, however, and most criticism has supposed that he adopted the first. His principal justification has been that no contradiction will be derived, using his axioms. Now it can be shown that the ordinary probability calculus can be developed fully using his axioms, and that in such a development no contradiction will ever be obtained—the axioms lead to a consistent set of rules of procedure. But absence of contradiction on such a level cannot be the main justification of a mathematical theory to any mathematician who believes his science is more than a chess-like game: surely a set of rules of procedure should have an acceptable base. What is desired is a mathematical theory which runs parallel to the physical facts, when properly idealized, but which has its own independent justification.

This edition of *Wahrscheinlichkeit Statistik und Wahrheit* contains a considerably enlarged critique of various theories of probability which will be of lasting value to all students of the subject.

J. L. Doob

Wahrscheinlichkeitsrechnung und allgemeine Integrationstheorie. By E. Tornier. Leipzig and Berlin, Teubner, 1936. 6+158 pp.

In the last few years, the theory of probability has been more and more influenced by the modern theories of measure. Professor Tornier gives a striking proof of this in devoting 100 of the 158 pages of his *Wahrscheinlichkeitsrechnung* to an interesting and fairly complete development of (Jordan) content and (Lebesgue) measure theories—treated from an abstract standpoint. The reader is warned in the introduction not to be deterred by this heavy array of pure mathematics: "so much mathematics is needed precisely in order to avoid reducing living basic intuitions into lifeless formalism, as results, for example, from an identification of probability with Lebesgue measure—inspired by the analogy in the rules of calculation." As we shall see, the author rejects Lebesgue measure in favor of Jordan measure, thus avoiding lifeless formalism.

Consider the theory of probability as applied to the analysis of the repeated casting of a single die, marked in the usual way. Any sequence $(n_1, n_2, \cdots)$ is logically possible, where n_j is one of the integers 1, 2, 3, 4, 5, 6. Tornier assigns probabilities to certain classes of these sequences. Thus to the class of all sequences for which $n_1=4$ (representing the possibility of casting a 4 the first time), is assigned the probability 1/6. More generally, if $a_1, \cdots, a_\nu$ is any finite set of integers between 1 and 6, the class of all sequences for which $n_j=a_j, j=1, \cdots, \nu$, is given probability $1/6^\nu$. These sets of sequences are called basic sets, and assigning these probabilities to the basic sets and prescribing the usual additive property of probability determines a probability measure—a set function defined on certain sets of sequences. This probability measure can be taken as (Jordan) content or (Lebesgue) measure, depending on the extent of the field of sets on which probability is defined. Now the author uses in a fundamental way special classes of sequences $(n_1, n_2, \cdots)$ having an intimate connection with the field of Jordan measurable sets determined by the basic sets, and this connection cannot be extended to the more general field of