ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

314. Professor R. P. Agnew: *On existence of linear functionals defined over linear spaces.*

A functional $r(x)$ defined over a linear space E is called an r-function (over E) if there exists a linear functional $f(x)$ with $f(x) \leqq r(x)$ for all $x \varepsilon E$. In order that $r(x)$ may be an r-function, it is necessary and sufficient that, for each $x \varepsilon E$, the greatest lower bound of $\sum_{k=1}^{n} r(t_k x_k)/t_k$, for all finite sets of positive numbers $t_1, \cdots, t_n$ and elements $x_1, \cdots, x_n \varepsilon E$ with $x_1 + \cdots + x_n = 0$, be non-negative. Moreover, if $r(x)$ is an r-function, then the greatest lower bound of $\sum r(t_k x_k)/t_k$, for all finite sets of positive numbers $t_1, \cdots, t_n$ and elements $x_1, \cdots, x_n \varepsilon E$ with $x_1 + \cdots + x_n = x$, is a p-function (Banach, *Théorie des Opérations Linéaires*, p. 28) $p^{(r)}(x)$ with $p^{(r)}(x) \leqq p(x)$; and $p^{(r)}(x)$ is the largest p-function with $p(x) \leqq r(x)$. (Received August 2, 1937.)

315. Mr. H. W. Alexander: *The role of the mean curvature in the immersion theory of surfaces.*

This paper falls naturally into two parts. In the first part an expression is obtained for the second fundamental tensor of a surface in terms of the mean curvature and its first and second derivatives, and the first fundamental tensor $g_{\alpha\beta}$ and its derivatives. These expressions are then substituted in the equations of Codazzi to obtain two third order differential equations in the mean curvature, which are completely integrable, and which are necessary and sufficient conditions that a quantity K_m be the mean curvature of a surface whose first fundamental tensor $g_{\alpha\beta}$ is given. In the second part, singularities of immersion of intrinsically regular surfaces are discussed, and are shown to be points where the mean curvature is infinite. Under a simple basic supposition, it is proved that the locus of such points is a regular curve E, "the edge of regression," and that the coordinates of the surface in the neighborhood of E are regular functions of the arc length along E and the square root of the arc length along geodesics orthogonal to E. Also, in the neighborhood of E there exists a second surface, applicable to the first and meeting it cuspidally. (Received July 31, 1937.)

316. Professor Reinhold Baer: *Groups with abelian central quotient group.*

The method employed throughout this investigation of groups which have abelian central quotient groups may be described as follows. If G is such a group, then preference is given to a subgroup S, situated between the central and the commutator group of G. Now G and S determine $G^* = G/S$ and certain invariant relations between the abelian groups G^* and S. The subgroups S and G^*, and these relations between G^* and S, are invariants of G if S is in particular either the central or the commutator group of G. In general these invariants do not completely determine the structure of G. They form a complete set of invariants, however, if G^* is a direct product of cyclic groups, and this is the only hypothesis restricting the generality of this method. (Received July 27, 1937.)

317. Professor Reinhold Baer: *Groups with preassigned central and central quotient group.*

It is well known that not every abelian group is the central quotient group of a suitable group, and those abelian groups with a finite number of generators which are central quotient groups have been characterized before. This note is devoted to a characterization of those pairs of groups C, Q which are central and central quotient group, respectively, of a suitable group. This problem is completely solved under the hypothesis that Q is a direct product of cyclic groups. The necessary and sufficient conditions for the existence of a group whose central is the abelian group C and whose central quotient group is the direct product Q of cyclic groups essentially relate the number of cyclic direct factors of Q whose order is n and the number of independent elements of order n in C. (Received July 27, 1937.)

318. Professor Harry Bateman: *Halley's methods for the solution of equations.*

About 1694 Edmund Halley gave two general methods for solving equations by iteration, one based on a rational expression for the second approximation in terms of the first and a second one in which the corresponding expression is irrational. It was this work that led to the discovery of Taylor's expansion of a function in a power series. In the present paper the convergence of the first process is considered, use being made of known results for the corresponding Newtonian process. Some extensions of the method are considered. (Received July 23, 1937.)

319. Dr. E. F. Beckenbach: *Remarks on the problem of Plateau.*

From the result that the problem of Plateau is solvable for an arbitrary finite-area-spanning Jordan contour, alternate proofs are given of the known result that the problem is solvable for an arbitrary Jordan contour. (Received July 17, 1937.)

320. Mr. Garrett Birkhoff: *Additive functions on Boolean algebras.*

Let B be any Boolean algebra, and let Σ denote the set of bounded additive real functions $f(x)$ defined on B. By the "total variation" of any $f\varepsilon\Sigma$, is meant the l.u.b. $|f|$ of the finite sums $\sum_{k=1}^{n}|f(x_k)|$, over disjoint elements $x_k\varepsilon B$. Relative to $|f|$ as norm, Σ is a Banach space, and the non-negative functions $p(x)$ with $p(I)=1$ form a closed convex bounded set Δ in Σ. Further, if T is a linear operator on Σ satisfying $T(\Delta)\leqq\Delta$, then $|T(f)|\leqq|f|$; that is, T is an isometry or a contraction. Moreover, if some iterate T^n of T satisfies $pT^n\geqq\lambda p_0$, $(\lambda>0)$, for all $p\varepsilon\Delta$, then T^n is a *uniform* contraction of Δ, and shrinks distances in ratios $\leqq(1-\lambda)$. It follows, generalizing a fixed-point theorem of Picard, that (1) there is exactly one $r_0\varepsilon\Delta$ left fixed by T, (2) powers T^k of T contract Δ to r_0 with the rapidity of a geometrical progression. If countable sums and products are defined in B, the completely additive functions $f(x)$ are a closed linear subspace Γ of Σ, and so the above results hold in Γ. Further, an argument of Hahn shows that, given $p_0\varepsilon\Delta$, the $f\varepsilon\Gamma$ for which $p_0(x)=0$ implies $f(x)=0$ are associated with *density* functions, whose integrals with respect to the measure p_0 are the original f. (Received July 10, 1937.)

321. Mr. Garrett Birkhoff: *On dependent probabilities.*

A matrix of transition probabilities is a linear operator T on the convex space Δ of "distribution functions." Hence if one introduces "total variation" as norm, transition probabilities can be interpreted geometrically. For example, if T is deterministic, it is isometric. Independent probabilities induce a projection onto a point. Moreover, in the "stochastic" case, $pT^n\geqq\lambda p_0$ for all p and some n, $\lambda>0$, p_0; hence iterations of T contract Δ towards a unique "stable" distribution—generalizing the usual theory of *discontinuous* stochastic processes. Further, each *continuous* stochastic process (that is, each continuously observed system whose transitions occur by occasional jumps) is associated with a continuously defined *rate of transition*, and conversely. In particular, each transition probability of change in a unit interval of time is associated (via the exponential-logarithmic series) with one and only one constant rate of transition. To establish this, one requires an infinitesimal calculus for linear operators on Banach space. (Received July 10, 1937.)

322. Professor E. T. Browne: *Sets of conjugate matrices.*

If M is a given square matrix of order n, and if $M_1, \cdots, M_{t-1}$ are $t-1$ matrices of the same order which possess the properties (1) the M's are commutative in pairs, and (2) $(\lambda-M)(\lambda-M_1)\cdots(\lambda-M_{t-1})\equiv f(\lambda)$, wherein λ is a scalar and $f(\lambda)$ a scalar polynomial in λ; then $M_1, \cdots, M_{t-1}$ are said to constitute a *set of conjugates* to M. If t is the smallest integer for which an identity of the above type holds, the set of conjugates is called a *reduced* set. Sets of conjugates have been studied by Taber, Franklin, Sokolnikoff, Pierce, Hermann, and others. In this paper a thorough study is made of sets of conjugates, and more general sets are obtained than those given hitherto. In particular, a special study is made of *cyclic* sets. (Received July 10, 1937.)

323. Dr. J. W. Calkin: *Self-adjoint differential operators of elliptic type. Resolvent kernels.*

The author defines in the complex Hilbert space $\mathfrak{L}_2(E)$, E an open set in the (x,y)-plane, a linear transformation T: $Tf=L(f)\equiv-(pf_x)_x-(pf_y)_y+qf$; p_{xx}, p_{yy}, and q bounded measurable, $p>0$, and $\mathfrak{J}(q)=0$ on every bounded closed set interior to E. The domain of T is characterized by explicit local conditions on its elements and the implicit condition $\int_E|L(f)|^2dE<\infty$. The following results are obtained: $T\equiv\tilde{T}$, $T\supseteqq T^*$. The deficiency-index of the symmetric transformation T^* is (n, n) and all of the cases $n=0, 1, 2, \cdots, \aleph_0$ occur. If H is a maximal symmetric extension of T^*, H^* its adjoint, and λ is a point of the resolvent set of H, there exist kernels $G(P, Q; \lambda)$, $G^*(P, Q; \bar{\lambda})$ of Carleman type such that, for every g in $\mathfrak{L}_2(E)$, $(H-\lambda I)^{-1}g(P)=\int_E G(P, Q; \lambda)g(Q)dQ$ and $(H^*-\bar{\lambda}I)^{-1}g(P)=\int_E G^*(P, Q; \bar{\lambda})g(Q)dQ$. The identity $\overline{G(Q, P; \lambda)}\equiv G^*(P, Q; \bar{\lambda})$ holds at every point of $E\times E$, $P\neq Q$. At the points $P=Q$, $G(P, Q; \lambda)$ becomes logarithmically infinite. The equations $L[G(P, Q; \lambda)]=\lambda G(P, Q; \lambda)$ and $L[G^*(P, Q; \bar{\lambda})]=\bar{\lambda}G^*(P, Q; \bar{\lambda})$, Q (P) fixed and $L(G)$, $L(G^*)$ calculated with respect to the coordinates of P (Q), are satisfied everywhere on $E-Q$ $(E-P)$. (Received August 3, 1937.)

324. Professor H. V. Craig: *On tensors relative to the extended point transformation.*

The principal object of this paper is to explain, by means of the concept extensor (tensor relative to the extended point transformation) and a process called contraction over a reduced range, the structure of certain of the invariants and vectors of higher order geometry. (Received July 27, 1937.)

325. Professor H. B. Curry: *Some properties of formal deducibility.*

In a deductive system $\mathfrak{S}$ let those intuitive propositions with whose deduction one is concerned in the formal developments be called elementary propositions; in the Hilbert theory, for example, these are of the form "$\mathfrak{A}$ is provable," where $\mathfrak{A}$ is a specific formula. Taking these propositions as basic, this paper is concerned with the system $\mathfrak{T}$ of propositions obtained from them by the usual logical sentence-combinations. These combinations are defined intuitively; for example, "$A \supset B$" means "B is provable in the system obtained by adjoining A to $\mathfrak{T}$." The problem of ascertaining what logical calculus is valid when its variables are interpreted as unspecified propositions of $\mathfrak{T}$ is considered. If the positive operations are the usual ones, and "not-A" is "A implies every elementary proposition," then the Heyting calculus is valid. With negation of another sort related to non-deducibility, the minimal calculus of Johansson with the addition (for certain $\mathfrak{S}$'s) of the law of excluded middle is valid. The notion of necessity is defined by considering a family of systems $\mathfrak{S}$; the resulting calculus bears a relation to the Heyting calculus similar to that which the calculus of C. I. Lewis bears to the classical one. (Received August 2, 1937.)

326. Dr. D. M. Dribin (National Research Fellow): *Normal extensions of quartic fields with the symmetric group.*

Let B be a normal algebraic number field of degree 6 whose group is the symmetric group on 3 letters. The author finds in the present paper necessary and sufficient conditions that $N=B(\mu_1^{1/2}, \mu_2^{1/2})$, ($\mu_1$ and μ_2 in B), be normal of degree 24 whose group is the symmetric group on 4 letters; these conditions are described by the effect of the autmorphisms of B upon μ_1 and μ_2 and are derived by simple class-field theoretic considerations. The author also proves the interesting result that every total-imaginary field B of the type described above can be extended to a field N of the type described with μ_1 and μ_2 taken as units in B. These results are applied to questions of existence of Hilbert subgroup series in normal extensions of quartic fields with the symmetric group, lately discussed by the author. (Received July 29, 1937.)

327. Professor O. J. Farrell: *On the representation of bounded analytic functions by sequences of polynomials.*

Let $f(z)$ denote a function analytic and bounded in a region G of the plane of the complex variable z. Let $d(f, P)$ denote the diameter of the cluster set of $f(z)$ at a point P on the boundary B of G. Let $D(f, B)$ denote the maximum of $d(f, P)$ for all points P on B. The problem considered is concerned with the existence of polynomials $p_n(z)$, $n=1, 2, \cdots$, which converge to $f(z)$ in G in such a way that as $n\rightarrow\infty$ the superior limit of the least upper bound of $[|f(z)-p_n(z)|, z \text{ in } G]$, does not exceed $D(f, B)$. A complete solution of the problem is not reached in this paper. It is found, however, that a sufficient condition for the representation of every such function by such a sequence of polynomials is that G be a limited Jordan region. Necessary conditions for the representation of every such function by such a sequence are that G be limited and, if simply connected, that B divide the extended plane into just two regions with B as their common boundary; if G is multiply connected, it is necessary that the closed region $G+B$ be the complement of an infinite region. (Received July 29, 1937.)

328. Dr. Aaron Fialkow: *Conformal transformations and the subspaces of a Riemann space.*

A conformal transformation, $d\bar{s}=\lambda ds$, which maps one Riemann space upon another induces a conformal correspondence between the respective subspaces of each. This induced mapping is studied in detail. A typical result is that two minimal varieties correspond if and only if the gradient $\partial\lambda/\partial x^i$ lies in the varieties. Other results do not depend upon the given conformal map but hold for a conformal Riemann space. Thus the lines of curvature always correspond. The hypersurfaces, $\lambda=$constant, play a dominant role in this work. Groups of conformal transformations (in particular, iterated maps) are also considered. (Received July 26, 1937.)

329. Dr. Kurt Friedrichs: *On the definition of differential operators.*

In order to apply the theory of self-adjoint linear operators to differential operators, it is not sufficient to consider functions which are differentiable in the ordinary sense. Instead of using concepts of Lebesgue's theory, the author gives a quite different method of extension, equally applicable to ordinary and partial differential operators and directly concerning those properties which are essential for self-adjointness. (Received August 3, 1937.)

330. Dr. H. H. Goldstine: *A multiplier rule in abstract spaces.*

The author formulates a problem of minimizing a functional, defined on a region of several normed linear spaces, in a class of points satisfying certain operational equations. This problem is designed so as to include the problem of Bolza in the calculus of variations, the problem of minimizing a function of several variables subject to side conditions, and numerous other special cases. Analogues of the Euler-Lagrange equations and of the transversality conditions are obtained by a method which is essentially a generalization of the ordinary proof given in the calculus of variations. (Received July 26, 1937.)

331. Dr. J. A. Greenwood: *Variance of a general matching problem.*

In this problem, t distinct groups of s identical cards each are matched against an equal-sized deck composed of $i_1, i_2, \cdots, i_t$ cards of each symbol. From a table giving the relative frequencies of hits and misses of all pairs of cards in the latter deck, the average correlation between all pairs of cards is found. Substitution in a well-known formula then gives the variance. It is also proved that the variance is a maximum when $i_1=i_2=\cdots=i_t=s$. Then $\sigma^2=n^2(t-1)/t^2(n-1)=n\sigma_b^2/(n-1)$ where $n=ts$ and σ_b^2 is the variance of the corresponding binomial case. (Received August 3, 1937.)

332. Mr. Philip Hall and Mr. J. K. Senior: *The groups of order* 64. Preliminary report.

The authors have made independently and by different methods a complete determination of the groups of order 64. Besides the 11 abelian groups, they have found 117 groups of class 2, 114 groups of class 3, 22 groups of class 4, and 3 groups of class 5. The total of 267 groups differs from that given in previous publications. A detailed account of the classification and properties of these groups is in course of preparation. (Received August 6, 1937.)

333. Dr. W. L. Hutchings: *On a canonical form of the system of differential equations defining a ruled surface.*

In this paper it is shown that if a ruled surface R_{yz}, defined by the system $y''+p_{11}y'+p_{12}z'+q_{11}y+q_{12}z=0$, $z''+p_{21}y'+p_{22}z'+q_{21}y+q_{22}z=0$, has distinct non-linear flecnode curves, the system may always be reduced to a canonical form for which $q_{12}=q_{21}=p_{11}+p_{22}=0$, $p_{12}=p_{21}=1$. Wilczynski's fundamental theorem that if the invariants $\theta_4, \theta_{4\cdot 1}, \theta_9, \theta_{10}$ are given arbitrarily $(\theta_4\theta_{10}\neq 0)$ they determine a ruled surface to within a projective transformation, is established without quadratures. The intersection of corresponding osculating planes of the flecnode curves of R_{yz} determine a ruled surface $R_{\eta\zeta}$ covariant with R_{yz}. The canonical form here obtained is applied to a study of the correspondences existing between these two surfaces. In this study, intersector curves, first defined by E. P. Lane, are of primary importance. At two points of each ruling of R_{yz} the asymptotic and intersector directions coincide. The invariant condition that the locus of these points be asymptotics, also that the involute curves of R_{yz} be asymptotics, is given. Conditions that the flecnode curves of the two surfaces correspond, that the flecnode curves of $R_{\eta\zeta}$ be plane curves, and that $R_{\eta\zeta}$ be a quadric surface are obtained. Some theorems of A. F. Carpenter concerning the flecnode and complex curves of a ruled surface are generalized and extended. (Received August 2, 1937.)

334. Dr. D. H. Hyers: *On functional equations in linear topological spaces.*

An existence theorem for differential equations in linear topological spaces has been proved by E. W. Paxson (see abstract 43-3-232), making use of the Brouwer-Tychonoff fixed point theorem. These theorems, however, are noneffective and tell us nothing about the uniqueness of the solution. In the present paper a new type of Lipschitz condition makes possible the use of successive approximations in proving existence and uniqueness theorems for differential and other functional equations in linear topological spaces. Point set theoretic methods are used extensively. An application is made to the theory of infinite systems of ordinary differential equations in real variables. (Received August 3, 1937.)

335. Professor M. H. Ingraham: *On certain equations in matrices whose elements belong to a division algebra.*

Roth, Franklin, the author, and others have studied the matrix equation $P(X)=A$, where P is a polynomial with coefficients in a given field. The natural generalization to matrices and polynomials with elements in a division algebra D leads to the study of the equation $P(X)\odot\xi=Q(A)\odot\xi\,(\xi)$, where the $\odot$-operation is defined as in an article by the author and Dr. M. C. Wolf (Transactions of this Society, vol. 42 (1937), pp. 16–31). The solution of this equation reduces to that of a system of equations with coefficients in the centrum. The solution of any one of these is found subject to factoring polynomials with coefficients in the centrum of D into irreducible factors with coefficients in D. The problem of finding the simultaneous solutions of such a system is still under consideration. (Received August 2, 1937.)

336. Dr. Nathan Jacobson: *A note on topological fields.*

Let F be a locally compact, separable, and totally disconnected (quasi-) field and J an anti-automorphism of F, that is, $(a+b)^J=a^J+b^J$, $(ab)^J=b^Ja^J$. Then F has degree one or two over its centrum C. If F has characteristic $\neq 2$ and J is involutorial ($a^{J^2}=a$ for all a in F) and $c^J\neq c$ for some c in C, then $F=C$ is commutative. Similar results hold for F_n a complete matrix ring with coefficients in F. An important lemma used in the proof is the following generalization of a result due to F. K. Schmidt: any automorphism or anti-automorphism of F is continuous (in fact isometric in a "naturally" defined metric). (Received July 19, 1937.)

337. Professor Fritz John: *On the solution of the differential equation* $u_{x_1x_1}+u_{x_2x_2}=u_{x_3x_3}+u_{x_4x_4}$.

In a recent paper in the Mathematische Annalen, L. Asgeirson proved for a solution $u(x_1, x_2, x_3, x_4)$ of this "ultrahyperbolic" equation the mean value theorem: $\int_0^{2\pi}u(x_1 + r\cos\phi,\ x_2 + r\sin\phi,\ x_3,\ x_4)d\phi=\int_0^{2\pi}u(x_1,\ x_2,\ x_3 + r\cos\phi,\ x_4+r \sin\phi)d\phi$ for any x_1, x_2, x_3, x_4, r. In the present paper u is interpreted as a function of the lines in xyz space with equations $x=x_1+x_3-(x_2-x_4)z$, $y=x_2+x_4+(x_1-x_3)z$. Asgeirson's theorem then states that for every hyperboloid of one sheet the values of the integrals of u over the two families of generating lines are equal. With the help of this theorem, it is proved that every solution of the differential equation satisfying certain regularity conditions may be represented as the line integrals of a function $f(x, y, z)$. (Received July 29, 1937.)

338. Dr. Gertrude S. Ketchum and Dr. P. W. Ketchum: *On a certain class of non-linear expansions of an arbitrary analytic function.*

It is shown that any function $f(x)$ analytic at the origin has the expansion $f(x)=f(0)+\sum b_nF_n(a_nx^n)$, where the functions $F_n(\omega)$ and the constants b_n are given in advance, subject to the following four conditions: (1) $F_n(\omega)$ has a simple zero at the origin, (2) $|F_n(\omega)| \leqq M_n$ for $|\omega| \leqq \rho$, where $M_n=O(n^a)$ for some a, (3) $|b_n|^{1/n}$ is bounded for all n, (4) there exist constants C, a, and σ such that $0\leqq|b_m| \leqq Cn^a\sigma^{n/m}|b_n|$ for all integers n and all integers $m<n$ which divide n. Certain infinite product expansions follow from this theorem. The particular case $F_n(\omega)=\omega(1-\omega)^{-1}$ has been studied by Feld (Annals of Mathematics, vol. 33 (1932), pp. 139–142). (Received July 29, 1937.)

339. Professors M. S. Knebelman and Walther Mayer: *Parallel vector fields.*

An n-dimensional affinely connected space is defined by n N-rowed square matrices $\Gamma^i_{j\alpha}(C^x_{x_0})$, $\alpha=1, \cdots, n$; $i, j=1, \cdots, N$, where $C^x_{x_0}$ is an arc joining the point x_0 to x. Here C is assumed to be piecewise differentiable and the functions Γ bounded, continuous, and independent of parametrization. A vector $y_0{}^i$ is displaced, parallel to itself, along C if $dy^i/dt=\Gamma^i_{j\alpha}y^jdx^\alpha/dt$; then $y^i=y_0{}^jF_j{}^i(C^x_{x_0})$ where the matrix F is given by an infinite series whose terms are multiple line integrals. The series is convergent and F is non-singular in some neighborhood of x_0. The fundamental property of F is $F(C^{x_1}_{x_0})F(C^x_{x_1})=F(C^x_{x_1})$. For the existence of N independent parallel vector fields the equations $\partial y^i/\partial x^\alpha=\Gamma^i_{j\alpha}y^j$ must be completely integrable. To obtain these conditions the problem is reduced to two dimensions and all curves are reduced to those made up of coordinate lines. By a limiting process the authors then express these conditions by the vanishing of a set of matrices whose elements are point functions. In the case of a connection whose matrices are point functions of class C', these reduce to the vanishing of the curvature tensor. (Received July 20, 1937.)

340. Professor R. E. Langer: *The expansion theory of ordinary differential systems of the first order.*

This paper deals with the differential system $y'(x) - [\rho q(x) + r(x)]y(x) = 0$, $\alpha y(a) = \beta y(b)$, in which the coefficients, except for the parameter ρ, are real, and in which $q(x)$ may change its sign. The theory for the expansion of arbitrary functions in solutions of such a system, as heretofore given (see M. H. Stone, Transactions of this Society, vol. 26 (1924), page 335), has been based on the orthogonality of the solutions over the interval (a, b). Its results are sharply in contrast with the analogous ones for differential systems of higher order. A new theory is here given. It is based on the orthogonality of the solutions over any point set Δ on (a, b), upon which the function $Q(x) \equiv \int_a^x q(x)dx$ fulfills, just once, the congruence $Q(x) \equiv t (\mathrm{mod}\ Q(b))$, for almost every t on the range $0 < t < Q(b)$. The results are much more nearly in agreement with those for other differential systems, and the necessary hypotheses on $q(x)$ are much less restrictive. These hypotheses are, in brief, that $q(x)$ be summable, that there exist an open point set Δ on which $Q(x)$ fills the requirement above, and on each component interval of which $q(x)$ is almost everywhere of one sign. (Received July 17, 1937.)

341. Dr. D. C. Lewis: *Invariant manifolds near an invariant point of unstable type.*

In the case of surface transformations the existence of an invariant curve in the neighborhood of an invariant point of unstable type and containing the invariant point was proved by Poincaré, Hadamard, and Birkhoff. Here the author gives a generalization of Hadamard's proof to the case of transformations in multiple dimensional spaces. (Received July 30, 1937.)

342. Dr. D. C. Lewis: *Representation of states in quantum mechanics by entire functions.* Preliminary report.

The impulse and coordinate operators p and q are replaced by the conjugate operators $x = p + iq$ and $y = p - iq$. Confining the problem for simplicity to the case of one degree of freedom, one is led to the representation of states by those entire functions $f(z)$ such that $|f(z) \exp(-z^2/(4h))|$ is bounded for real z and $|f(z) \exp(z^2/(4h))|$ for pure imaginary z. Here $2\pi h$ is Planck's constant. A certain subclass of this class of entire functions forms a Hilbert space, in which the inner product of two elements is defined in terms of the coefficients in the power series of the corresponding two entire functions. Any complete set of eigenstates (including those of p and q themselves) which do not lie in a Hilbert space are "relatively" normalized with respect to each other by the requirement that the corresponding representatives of p and q be Hermitian. A general theorem to the effect that an arbitrary entire function of the type indicated can be represented in the form $\int_{-\infty}^{+\infty} \phi(z, \lambda) d\alpha(\lambda)$, where $\phi(z, \lambda)$ represents an eigenstate of an assigned Hermitian operator with eigen-value λ, has not been obtained. Upon such an expansion a probability theoretic interpretation may be built up in almost the usual way. (Received July 30, 1937.)

343. Mr. L. L. Lowenstein: *Linear equations with an infinity of unknowns.*

This paper establishes criteria that a set of equations $\sum_{k=1}^{\infty} a_{pk}x_k = c_p$, $p=1, 2, \cdots, m \leqq \infty$, either have solutions (1) such that for each $\epsilon>0$, $\sum|x_k|^2<\epsilon$ or (2) such that for all solutions $\sum|x_k|^2 \geqq \theta > 0$. For $m<\infty$ let $\{a_{1k}\}, \cdots, \{a_{mk}\}$ be a set of linearly independent sequences of numbers. If there exist numbers $\mu_1, \cdots, \mu_m$, not all zero, such that $\sum_{k=1}^{\infty}|\sum_{p=1}^{m}\mu_p a_{pk}|^2<\infty$, then the set of sequences will be said to be *quadrilinearly dependent*; otherwise *quadrilinearly independent.* For $m<\infty$ quadrilinear independence and dependence of the set of sequences of coefficients of the equations under consideration are criteria for solutions of type (1) and type (2), respectively. If $m=\infty$ then it is said that the set of sequences is quadrilinearly dependent if some finite subset is quadrilinearly dependent. Quadrilinear independence is a necessary condition for solutions of type (1) if $m=\infty$. It is not sufficient; however, it is sufficient to ensure the existence of a certain solution in a generalized sense of type (1). Necessary and sufficient conditions for the quadrilinear dependence of sets of sequences are established. (Received August 3, 1937.)

344. Professor R. G. Lubben: *Perfectly compact Hausdorff spaces in which a normal space may be embedded.*

Let S be a normal Hausdorff space and let M be the aggregate of all T's, where T is a perfectly compact Hausdorff space of which S is a sub-space and in which S is dense. The author defines the term "boundary point of S" and establishes the existence of a sub-aggregate N of M such that (1) if T_N is an element of N, the points of T_N-S are boundary points of S and (2) each element of M is topologically equivalent to some element of N. There exists an element T_ω of N such that (1) the points of $T_\omega-S$ are "atomic" boundary points of S, and (2) if T is an element of M, there exists in T_ω an upper semi-continuous collection, K_T, of mutually exclusive closed point sets, the points of S being elements of K_T, such that T is topologically equivalent to the decomposition space of T_ω which is determined by K_T. (Received July 10, 1937.)

345. Mr. J. K. L. MacDonald: *Bounds for parameters in n-noded solutions of Sturm-Liouville equations.*

The form $dp(x)/dx\cdot dy(x)/dx+q(x)y(x)=0$, $(p(x)>0,\ q(x)>0$ in $(x_1, x_2))$, is considered with respect to the relations between parameters in $q(x)$, and the number of nodes for $y(x)$ in (x_1, x_2). A first order differential equation for a phase angle is developed by use of a transformation involving two adjustable functions. By suitable choices of the adjustable functions in various cases, the first order equation is expressed in forms convenient for approximate integration (with bounds for errors). Means for including end conditions are also developed. (Received July 30, 1937.)

346. Dr. A. J. Maria: *Displacement of equilibrium point of Green's function for an annulus.*

In this note there is given an estimate of the displacement of the equilibrium point of Green's function for an annulus as the pole moves along a fixed diameter from the inner to the outer boundary. There are also results on the magnitude of the displacement for small and large annuli. (Received August 6, 1937.)

347. Professor W. H. McEwen: *A note on an extension of Bernstein's theorem.*

In a recent paper (American Journal of Mathematics, vol. 59 (1937), pp. 295–305) the author obtained an extension of Bernstein's theorem to arbitrary sums of Birkhoff type, of the form $S_N(x)=\sum_{i=1}^{N}a_iu_i(x)$ in which the u's are the first N characteristic solutions of a general regular nth order linear differential system $L(u)+\lambda u=0$, $W_i(u)=0$, $(i=1, 2, \cdots, n)$, defined on $a\leqq x\leqq b$. Assuming that $|S_N(x)|\leqq L$ on (a, b), the author found that $|S_N'(x)|\leqq QNL$ uniformly on any interior interval $(a+\delta, b-\delta)$, Q being a constant independent of N, and an example was cited to show that this is the best result that can be obtained in general. In particular cases, however (as for example in the Fourier case), the limit QNL can be extended to the whole interval (a, b). In this paper general additional hypotheses are considered under which this extension to the whole interval can be made. (Received July 26, 1937.)

348. Professor A. D. Michal: *General conformal differential geometry.*

General Riemannian differential geometry with Banach coordinates has already been studied by the author on several occasions. The theory centering around a conformal curvature form is developed in the present paper with the aid of additional postulates. (Received August 3, 1937.)

349. Professor A. D. Michal: *General projective differential geometry.*

This paper deals with a new chapter in the author's general differential geometries with Banach coordinates. The normed ring of linear transformations in the Banach space is subjected to postulates insuring the existence of a suitable "contraction." The existence of a projective curvature form is demonstrated and it is then proved (barring a few singular cases) that a necessary and sufficient condition that a general geometry with a symmetric linear connection be locally projectively flat is that the projective curvature form vanish. The existence theorems for completely integrable abstract differential equations play a fundamental role in these studies. (Received August 3, 1937.)

350. Dr. A. P. Morse and Dr. J. F. Randolph: *Gillespie linear measure for point sets.*

Several definitions of the linear measure for a point set A have been given. Of these linear measures, the one $L(A)$ by C. Carathéodory seems to be the most adequate generalization of curve length. In this paper, however, a plane set A is constructed such that its projection X on the x-axis is $0\leqq x\leqq 1$ and Y is $0\leqq y\leqq 1$, but nevertheless $L(A)$ is only unity instead of at least $2^{1/2}$. A different linear measure $G(A)$ is defined and named Gillespie linear measure after the late Professor D. C. Gillespie who suggested similar definitions to the authors. With $|X|$ the outer Lebesgue measure of X, it is proved for any plane set A and any axes that $G(A) \geqq (|X|^2+|Y|^2)^{1/2}$. If A consists of all points of a simple rectifiable arc, the length of the arc and $G(A)$ are proved equal. For such sets $G(A)=L(A)$. Always $L(A) \leqq G(A)$ and it is easily shown that $G(A) \leqq \pi L(A)$. Furthermore it is proved that $G(A) \leqq (\pi/2)L(A)$. This relation is shown to be the best possible by constructing a set for which the equality sign holds. Also, Gillespie area measure is defined for sets in three or more dimensions and a basic relation between "G-area" and "G-length" is established, the analogue of which has not been proved for Carathéodory's measures. (Received August 3, 1937.)

351. Dr. D. L. Netzorg: *An extension of Markoff's comparison theorems for zeros of orthogonal polynomials.*

Let $\rho_1(t) < \rho_2(t) < \cdots < \rho_n(t)$ be the zeros of the nth polynomial $P_n(x, t)$, orthogonal with respect to the non-negative weight function $p(x, t)$ over a fixed interval $a \leqq x \leqq b$. If $(1/p)\partial p/\partial t$ is a convex function of x over (a, b), it is impossible to have $i<j$ while $\rho_i(t)$ is an increasing, and $\rho_j(t)$ a decreasing function of t. If, furthermore, $a=-h$, $b=+h$, and p is an even function of x over (a, b), $\rho_i^2(t)$ is an increasing function of t ($i=1, 2, \cdots, n$, except for $i=m$, if $n=2m-1$, where $\rho_i(t)=0$), so that the positive zeros increase, and the negative zeros decrease, as t increases. Taking $p(x, t)=(h^2-x^2)^t p(x)$, where $p(x)$ is an even function over $(-h, +h)$, one obtains a theorem due to Stieltjes (Acta Mathematica, vol. 9 (1886), pp. 385–400). Markoff's proof of the same theorem (Mathematische Annalen, vol. 27 (1886), pp. 177–182) is erroneous, as Professor Szegö has pointed out. (Received August 4, 1937.)

352. Dr. D. L. Netzorg: *Certain inequalities for functions of exponential type belonging to L along the real axis.*

If the entire function $f(z)=O(e^{|z|})$ is such that $f(x)$ belongs to $L_2(-\infty, +\infty)$, then, for every $\alpha \geqq 0$, $\int_{-\infty}^{+\infty}|t^\alpha f(t)|dt \geqq C_\alpha |\int_{-\infty}^{+\infty} f(t)dt|$, where $C_\alpha > 0$ depends only on α. (Received August 4, 1937.)

353. Dr. D. L. Netzorg: *The mean number of solutions of $\phi(x)=n$.*

Denote by $g(n)$ the number of solutions of $\phi(x)=n$, where ϕ is the Euler ϕ-function. Then $\lim_{n\to\infty}(1/n)\sum_{\nu=1}^{n} g(\nu)=\prod_p(1+1/p(p+1))=1.9432+$, the product extending over all primes. Similar asymptotic evaluations are obtained for $\sum \nu^a \phi^b(\nu)$ summed over $\nu^\alpha \phi^\beta(\nu) \leqq n$. Generalizations are given to any numerical function $\psi(n)$ such that $\psi(mn)=\psi(m)\psi(n)$ when $(m, n)=1$, and $\psi(p^k)=p^{k-1}\psi(p)$ for all primes p. A recurrent method is given for constructing a table of solutions of $\phi(x)=n$. (Received August 4, 1937.)

354. Professor E. G. Olds: *The distribution of sums of squares of rank differences for the case of eight individuals.*

In a previous paper (abstract 43-1-48), the author presented the exact distributions for sums of squares of rank differences for the cases of numbers of individuals from two to seven, and compared them with approximations obtained by use of the Gaussian and Pearson type II functions. The present paper gives the exact distribution for the case of eight individuals and indicates the degree of accuracy when the exact distribution is replaced by one of the above-mentioned functions in testing the significance of rank correlation. (Received July 30, 1937.)

355. Dr. A. E. Pitcher and Dr. W. E. Sewell: *Existence theorems for solutions of differential equations of non-integral order.*

Liouville and Riemann considered a derivative in which the order α may assume any real value. In this paper the authors establish an abstract existence theorem of the sort proved by Birkhoff and Kellogg. It is applied to prove existence theorems for solutions of differential equations of the form $y^\alpha = \phi(x, y)$. In the above equation α is positive, ϕ is a known function of two real variables, y is an unknown function of x, and y^α denotes the derivative of order α. (Received August 3, 1937.)

356. Professor Tibor Radó: *On continuous transformations of bounded variation in the plane.*

Let T: $x = x(u, v)$, $y = y(u, v)$ be a continuous transformation, defined in the square Q_0: $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$. If T is of bounded variation, then it possesses a certain area-derivative $D(u, v)$ which exists almost everywhere in Q_0 and is summable there (see Banach, Fundamenta Mathematicae, vol. 7 (1925), pp. 225–236). Schauder (Fundamenta Mathematicae, vol. 12 (1928), pp. 47–74) introduced a topologically defined function $i(u, v)$ and derived a number of results which show that the product $J(u, v) = i(u, v)D(u, v)$ may be considered as a generalization of the Jacobian of a pair of functions. In order to secure the summability of $J(u, v)$, Schauder (loc. cit.) assumed that $i(u, v)$ is bounded. In a previous paper (Fundamenta Mathematicae, vol. 27 (1936), pp. 201–211), the present author proved that $J(u, v)$ is always summable if the transformation T is absolutely continuous in the sense of Banach. In the present paper, it is shown that this result remains true for transformations of bounded variation in the sense of Banach. (See Banach, loc. cit., for the terminology.) (Received July 23, 1937.)

357. Professor J. F. Ritt: *On certain points in the theory of algebraic differential equations.*

The following topics are treated: forms in several unknowns; pairs of forms; essential manifolds composed of one solution; an approximation theorem; equations in two unknowns, of the first order. (Received July 28, 1937.)

358. Mr. L. B. Robinson: *A quasi-analytic function which satisfies a functional equation.*

A generalized Lambert series has been studied (M. C. Garvin, American Journal of Mathematics, vol. 58 (1936), pp. 507–513), giving an example of a quasi-analytic function. Consider the series $f \equiv F_0 + \lambda F_1 + \cdots + \lambda^n F_n + \cdots$ wherein $F_n \equiv \int_0^\xi (1-r^2)^{-1} \int_0^r (1-s^4)^{-1} \cdots \int_0^y (1-z^{2^n})^{-1} dz^{2^{n-1}} dy^{2^{n-2}} \cdots ds^2 dr$. The function f has a natural boundary on the unit circle. Since f also converges elsewhere for sufficiently small values of λ, f is quasi-analytic. It also satisfies the functional equation $u'(\xi) = \lambda u(\xi^2)/(1-\xi^2)$. (Received April, 1937.)

359. Professor A. E. Staniland: *Analytic transformations in euclidean space of 2n dimensions.*

A euclidean $2n$-space, S_{2n}, is employed as a representation of a complex n-flat, Σ_n, coordinates of mated points of the two spaces subject to the correspondence $z_k = x_{2k-1} + ix_{2k}$, $(k = 1, 2, \cdots, n)$. Sets of analytic functions defined in Σ_n are represented by analytic hypersurfaces S_{2n}. An analytic transformation of S_{2n} is one that leaves invariant the class of analytic surfaces. A transformation $x_r = f_r(x_1', x_2', \cdots, x_{2n}')$, $|\partial f_r/\partial x_s'| \neq 0$ and $\partial f_r/\partial x_s'$ continuous, is analytic if and only if it is equivalent to one of the two analytic types, $A: z_k = \phi_k(z_1', z_2', \cdots, z_n')$, $B: \bar{z}_k = \phi_k(z_1', z_2', \cdots, z_n')$. The jacobians, j and J, of the real and complex transformations are related by the identities $j \equiv \pm J\bar{J}$, the negative sign holding for B(n odd). Types A and B are *normal* at P_0 in Σ_n when the roots of $[\partial\phi_k/\partial z_j']$ are all distinct at P_0. Then the *differential* transformations A_1 and B_1, of matrix $[\partial\phi_k/\partial z_j']$, regarded formally as transformations of the neighborhood of P_0 into itself, leave invariant n complex line elements E_1^μ through P_0. With the E_1^μ as new complex axes, A_1 and B_1 assume the multiplicative forms $dz_\mu = r_\mu e^{i\theta_\mu} dz_\mu'$ and $d\bar{z}_\mu = r_\mu e^{i\theta_\mu} dz_\mu'$. The latter reduces to $d\bar{z}_\mu = r_\mu dz_\mu'$ when the new axes are used. The E_1^μ correspond to n real invariant plane elements Π_μ through Q_0, the representative of P_0; hence the mapping of S_{2n} upon itself by A and B is conformal with respect to the *components* in Π_μ of elementary arcs originating at Q_0. (Received August 7, 1937.)

360. Dr. A. H. Taub: *Spin representation of inversions.*

A subgroup of the conformal group of a flat n-space is the direct product of the rotation group and the group of two elements composed of the inversion and the identity. The spin representation of the former is well known. The spin representation of the latter is obtained. The representation is then used to study the behavior of Dirac's equation under conformal transformations. The results of Dirac and Veblen are obtained from this viewpoint. It is further shown that if the mass term in Dirac's equation is set equal to zero, the equation is conformally invariant. Since this is done in the neutrino theory of light, it is thus proved that this theory has the same invariance property as the Maxwell theory. (Received July 26, 1937.)

361. Dr. A. H. Taub: *Spin representations of groups of motion of Riemannian spaces of constant curvature.*

Riemannian spaces of constant curvature can be imbedded in flat spaces of one higher dimension. The group of motion of the former is then represented by a subgroup of the euclidean group of the flat space. The spin representation of the latter is known, and hence one obtains a spin representation of the group of motion of any Riemannian space of constant curvature. In obtaining the representation explicitly, it is found that two coordinate systems are equally useful. The relation between these is also discussed. (Received July 26, 1937.)

362. Dr. R. M. Thrall and Dr. Josephine H. Chanler: *Ternary trilinear forms in the field of complex numbers.*

Equivalence and general equivalence are defined for the general trilinear form $\sum_{h=1}^{k}\sum_{i=1}^{l}\sum_{j=1}^{m} a_{hij}x_h y_i z_j$, where k, l, m are the smallest numbers of x's, y's, and z's in terms of which the form can be expressed, and a_{hij} are complex numbers. In *Metabelian groups and trilinear forms* (presented to the Society, April 9, 1937), Thrall has shown that the problem of determining equivalent classes of forms is abstractly identical with that of classifying the matrices $M_x(x) = (\sum a_{hij}x_h)$ under certain multiplications and transformations; in that article the classification was carried out for the ternary case for forms in a $GF(p)$. In the present paper the main problem is the determination of generally-equivalent classes of ternary forms in the complex field. Here a geometric method is more illuminating. The conditions imposed by the form upon the curves $|M_x(x)| = 0$, $|M_y(y)| = 0$, $|M_z(z)| = 0$, are studied; in particular, the projective invariants of the curves are shown to be invariants of the form, and certain Cremona transformations relating them are investigated. A table of generally-equivalent classes is given, and a typical member of each class is exhibited. (Received July 30, 1937.)

363. Mrs. C. C. Torrance: *Superposition on monotonic functions.*

In this paper the following theorem is proved. Let $g(x)$ be a general monotonic function. Then if $f(y)$ has the Baire property in the restricted sense, $f[g(x)]$ has the Baire property in the restricted sense. Whether or not $f(y)$ has the Baire property, $f[g(x)]$ can (1) have the Baire property in the restricted sense, or (2) have the Baire property, or (3) fail to have the Baire property. (Received July 16, 1937.)

364. Professor W. J. Trjitzinsky: *Theory of non-linear singular differential systems.*

In this work the author investigates (in the complex plane, in the vicinity of the singular point $t=\infty$) properties of solutions of systems (A) $t^{-p}y_j^{(1)}(t) = a_j(t, y_1, \cdots, y_n)$, $j=1, \cdots, n$, integer $p \geqq 0$, wherein second members are analytic in $t, y_1, \cdots, y_n$ at $(t=\infty, y_1=0, \cdots, y_n=0)$. The particular case of (A) when the second members are independent of t and $p=0$ is of importance in dynamics. This problem, called (B), has been also investigated at length in the complex t-plane. Finally, systems (C) $\lambda^{-p}y_j^{(1)}(x) = a_j(\lambda, x, y_1, \cdots, y_n)$, $j=1, \cdots, n$, integer $p \geqq 0$ (λ a complex parameter; x on a real interval Δ), have been studied under the hypothesis that the second members are analytic in $\lambda, y_1, \cdots, y_n$ at $(\lambda=\infty, y_1=0, \cdots, y_n=0)$ for x in Δ, differentiable in x. The investigation for (C) has been given for the vicinity of the singular point $\lambda=\infty$. The author's papers on linear differential equations in the Acta Mathematica (vol. 62, pp. 167–226; vol. 67, pp. 1–50) were of much utility in this investigation. A specific method recently developed by the author for non-linear problems (cf. a paper in Compositio Mathematica, vol. 5, pp. 1–66, and a forthcoming work in the Mémorial des Sciences Mathématiques) has been widely used in this work. The present paper will appear in the Transactions of this Society. (Received July 31, 1937.)

365. Professor A. W. Tucker: *Degenerate cycles bound.*

In topological discussions involving singular chains it has been customary to suppress the degenerate elements which necessarily arise from the use of transformations of a simplicial nature. The following elementary argument shows that this suppression is unnecessary for purposes of homology. Any degenerate cycle on a simplicial complex can be expressed as a sum of cycles eC, where e is a degenerate simplex each vertex of which is repeated an even number of times and C is a wholly non-degenerate cycle. But $aeC \rightarrow eC$ where a is a vertex of e. Hence the degenerate cycles fall in the group of bounding cycles. (Received August 4, 1937.)

366. Dean G. D. Birkhoff: *An extensive class of analytic functions.*

This paper will appear shortly in the Annales de l'Institut Henri Poincaré. (Received August 31, 1937.)

367. Professor O. E. Glenn: *The generalized law of Bode as a mathematical principle.*

Bode's law, which expresses the mean distances (r) of our planets from the sun by the formula $ar = z^\lambda + b$, was known to Kepler and to Bode in the special, inaccurate form $a = 1/3$, $b = 4/3$, $z = 2$, $\lambda = 0, 1, \cdots, 7$. The author gives the first justification, on theoretical grounds, of the general formula as related to central systems. The mathematical principle reaches the following degree of generality. Let $f(\cdots, d^n r/d\theta^n, \cdots) = 0$, $(n = i, i-1, \cdots, 0)$, be an equation rational and integral in r and in the derivatives, and otherwise numerical and real. Suppose $\theta = q(r_1, r_2) \equiv \alpha r_1^m + \beta r_1^{m-1} r_2 + \cdots + \kappa r_2^m$ represents, parabolically, integral curves in a finite region, and let $\phi(r_1, r_2)$ be a covariant of q under the general linear, real transformation T. One can transform $f=0$ rationally so that $\phi(r, 1)$ becomes the dependent variable. If s is an integral curve of the new equation and σ the radial coordinate of a point of s, the equation $\phi(r, 1) = \sigma$ is invariant under cases T_j of T. This fact gives information concerning the roots, that is, the intersections, with s, of the integral curves of $f=0$. In the example where ϕ is the central force for stable orbits, T being special, the roots of $\phi = \sigma$ are a sequence $t, zt, z^2t, \cdots, z^kt, \cdots$, which may be identified with the sequence of Bode. (Received August 21, 1937.)

368. Professor E. L. Dodd: *Some properties of the general or substitutive mean.*

The function $F(t_1, t_2, \cdots, t_k, u_1, u_2, \cdots, u_n)$ will be called a mean of the u's if for each set of the t's and c for which F is defined, $F(t_1, t_2, \cdots, t_k, c, c, \cdots, c) = c$, it being assumed that there is at least one such set. This mean is transitive; but it need not be internal, translative, homogeneous, symmetrical, or associative. Let $f(x_1, \cdots, x_n, y_1/x_1, \cdots, y_n/x_n)$ be defined for all sets of x's none of which is zero, and let f be a mean of the ratios y_i/x_i. Let $\overline{X} = \phi(X_1, \cdots, X_n)$ be a single-valued homogeneous mean of $X_1, \cdots, X_n$, such that for at least one set of X_i's, $X_i-\overline{X}\neq 0$. (Homogeneity requires that $\phi(cX_1, \cdots, cX_n)=c\phi(X_1, \cdots, X_n)$. And let $\overline{Y}=\phi(Y_1, \cdots, Y_n)$. When $X_i\neq 0$ and $X_i-\overline{X}\neq 0$ define the function g as follows: $g(X_1, \cdots, X_n, Y_1/X_1, \cdots, Y_n/X_n) = f\{X_1-\overline{X}, \cdots, X_n-\overline{X}, (Y_1-\overline{Y})/(X_1-\overline{X}), \cdots, (Y_n-\overline{Y})/(X_n-\overline{X})\}$. Then g is a mean of the ratios Y_i/X_i. (Received August 31, 1937.)

369. Dr. Aaron Fialkow: *Hypersurfaces of a space of constant curvature.*

The following characterization of spaces of constant curvature by means of their hypersurfaces is proved: When none of the lines of curvature at a point of a hypersurface V_n of a V_{n+1} of constant curvature is tangent to a null vector, their tangents are Ricci principal directions for the V_n. Conversely, if every hypersurface of a $V_{n+1}(n\geqq 3)$ has this property, the V_{n+1} is a space of constant curvature. For $n=2$, every surface V_2 in any Riemann space V_3 has the above property. The paper also gives a complete classification of the Einstein spaces V_n which may be imbedded in a V_{n+1} of constant curvature so that none of the lines of curvature of V_n is tangent to a null vector. This generalizes a recent theorem of T. Y. Thomas and E. Cartan. (Received August 27, 1937.)

370. Dr. Caroline A. Lester: *A determination of the automorphisms of certain algebraic fields.*

In this paper all automorphisms are explicitly obtained as rational functions of one root of the defining equation for the cyclic cubic, quartic, and sextic, the quartic with the four-group, the normal sextic with the symmetric group, and the octics with the abelian groups of types (2, 2, 2) and (4, 2). The determination of the parametric representations of the most general equations defining these fields forms an integral part of the determination of the automorphisms. The computation in the cases $n=6$ and $n=8$ is brought within practicable limits by a free use of matric theory, in particular of a theorem of Williamson (this Bulletin, vol. 37 (1931), p. 585). This has been used in the concluding section to obtain from the results of Albert (Transactions of this Society, vol. 35 (1935), p. 949) a one-parameter family of cyclic octics over the rational field together with their automorphisms. (Received August 30, 1937.)

371. Mr. R. E. O'Connor: *The quaternion congruence* $\bar{\tau}\alpha\tau\equiv\beta$ (mod p).

Let $\alpha=[a_1, a_2, a_3]$ and $\beta=[b_1, b_2, b_3]$ be pure, integral quaternions; p an odd prime; p dividing $N\alpha$ and $N\beta$ but not α, β. Gordon Pall has shown (see abstract 43-7-303) that the congruence $\bar{\tau}\alpha\tau\equiv\beta(p)$ is, for given α, solvable in integral quaternions τ for precisely $(p^2-1)/2$ (non-zero) incongruent quaternions β. A criterion is now given for these residues β. By expanding $a_i(\bar{\tau}\alpha\tau)_i$ and setting $g=a_1b_1+a_2b_2+a_3b_3$, it is shown that β must satisfy either C_1, $p|g$, $(2g|p)=1$; or C_2: $p|g$, $\beta\equiv m\alpha(p)$, $(m|p)=1$, m being a rational integer. The condition is shown to be sufficient by counting simultaneous (non-zero) solutions β of the congruences $N\beta\equiv 0(p)$, $g\equiv r(p)$, r any rational integer, which yield p or $p-1$ solutions according as r is or is not $\equiv 0(p)$. It follows easily that, if $p|g$, $\beta\equiv s\alpha(p)$ with s a rational integer, precisely $(p^2-1)/2$ quaternion residues β(pure, non-zero) satisfy C_1 or C_2. (Received August 30, 1937.)