ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

287. C. R. Adams and J. A. Clarkson: *The type of certain Borel sets in several Banach spaces.*

Let AC represent the class of functions $x(t)$ absolutely continuous on $0 \leqq t \leqq 1$, CBV the class of continuous functions of bounded variation, C the class of continuous functions, R the class of properly Riemann integrable functions, and R^* the class of properly or improperly Riemann integrable functions. Oxtoby (see this Bulletin, vol. 43 (1937), pp. 245–248) has shown that the subsets C, R, and R^* of each space $L_p([0, 1])$, $(p \geqq 1)$, are $F_{\sigma\delta}$ sets of first category. In the present paper, the determination of their Borel type is completed by showing that each is no $G_{\delta\sigma}$. It is proved also that the subset AC of each of the spaces C, L_∞ (or M), and L_p is likewise an $F_{\sigma\delta}$ but no $G_{\delta\sigma}$, and that $CBV \subset L_p$ is both an $F_{\sigma\delta}$ and a $G_{\delta\sigma}$ but neither an F_σ nor a G_δ. The chief tool employed is a characterization of a G_δ in any metric space which runs roughly as follows: E is a G_δ if and only if no sequence $\{x_n\} \subset E$ which converges *rapidly enough* tends to a limit outside of E. For resolving questions of the sort here considered, the range of applicability of the methods mainly employed is by no means restricted to Banach spaces. (Received May 4, 1938.)

288. G. E. Albert: *Asymptotic forms for the generalized Legendre functions.*

Complete asymptotic forms for the functions $P_n{}^m(z)$ and $Q_n{}^m(z)$, n, m, and z complex, are derived under the following sets of conditions: (1) n is large, m is relatively moderate, and (a) z is exterior to some small neighborhoods (depending on n) of the critical points $z = \pm 1$, or (b) z is within such a neighborhood; (2) m is large, n is relatively moderate, and (a) $|z| < c|m|$, or (b) $|z| \geqq c|m|$, is valid for the smaller of the two indices in any bounded domain. The cases (1a) and (2a) are subject to Stokes' phenomenon; the discontinuities incurred are studied completely. The formulas developed corroborate all previously known results. The results for case (1b) are almost entirely new, only a few special cases of them having been studied before. Those for case (2b) are totally new. The method of the paper consists in an identification of the associated Legendre equation with a more general equation whose asymptotic solutions have been given by R. E. Langer in terms of elementary functions or Bessel functions. The interdependence of solutions leads to the calculation of the desired forms. (Received May 19, 1938.)

289. J. J. De Cicco: *Additional properties of the second derivative of a polygenic function.*

This is a continuation of the paper, *The second derivative of a polygenic function,* by Kasner (Transactions of this Society, vol. 30 (1928), pp. 803–818). Kasner has shown that the second derivative $\sigma = d^2w/dz^2$, taken over an arbitrary path of approach to the *fixed* point z, defines a point transformation Σ between the centers of curvature $Z = X + iY$ relative to the fixed point $z = x + iy$, of the Z-plane, and the points of the σ-plane. Kasner proved that there is a fundamental cardioid E in the σ-plane such that the correspondence between the points in the Z- and σ-planes is (1, 1) inside E, (2, 1) on E, and (3, 1) outside E. The ∞^1 lines through z are converted into the ∞^1 lines tangent to E. The present paper shows that the ∞^2 straight lines not through z are converted into a congruence Γ of limaçons. In Γ there is a unique circle C_M. This corresponds to two lines L_C and L_C'. The correspondence between all the other lines and limaçons of Γ is (1, 1). In Γ there are ∞^1 cardioids G. These have concentric base circles and their poles are on E. The minimum cardioid is E, and there is one maximum cardioid M. Its base circle is C_M. If L_E and L_M are the lines which are transformed into E and M, then L_E, L_M, L_C, L_C' are concurrent, and L_E, L_M separate L_C, L_C' harmonically. Finally, the ∞^1 lines which are transformed by Σ into the ∞^1 cardioids G envelop a conic. (Received May 16, 1938.)

290. J. J. De Cicco: *Asymptotic direction of a field of lineal elements.*

This is a continuation of the paper, *Geometry of turbines, flat fields, and differential equations,* by Kasner and the present author, and abstract 43-5-260 by the author, *The differential geometry of series of lineal elements.* Let F be an element in the field adjacent to a given element E, and let ER be the turbine of intersection of the tangent flat fields at E and F. The limiting turbine directions of EF and ER are called *reciprocal* at E. Theorem 1: Let T_0 be the principal tangent turbine (the extremal general curvature direction), and let T and T' be two tangent turbines at E. Let S and S' be the distances of T and T' from T_0. A necessary and sufficient condition that the directions T and T' be reciprocal is $SS' = -K_0/\lambda$, where K_0 is the extremal general curvature and λ is the equi-parallel curvature. Self-reciprocal directions are called asymptotic directions. Theorem 2: For a series to be an asymptotic series, it is necessary and sufficient that its osculating flat fields coincide with the tangent flat fields. Theorem 3: A necessary and sufficient condition that a series be an asymptotic series is $\tau = \pm G^{1/2}$, where τ is the torsion of the series and G is the curvature of the field (the analogue of the Beltrami-Enneper theorem). (Received May 16, 1938.)

291. F. C. Gentry: *Cremona involutions determined by a pencil of quartic surfaces.*

A projective correspondence is established between the members of a pencil of quartic surfaces containing a double line d and the points of d. A line through a point of d meets the surface of the pencil corresponding to that point in two points which are in involution. The special configurations of the F-curves of the second kind of the transformation, which arise when the residual basis curve of the pencil of surfaces is composite, are determined. (Received May 23, 1938.)

292. F. C. Gentry: *Quaternary Cremona groups of ternary type.*

The webs of quartic surfaces of degree two which contain a double curve and one or more simple points in their bases are determined. Each such web of surfaces determines an involution in space. The involutions obtained by allowing the simple base-points of such a web of surfaces to vary while the remainder of the base is held fixed are used to generate groups of Cremona transformations. These groups are shown to be simply isomorphic to the linear groups $g_\rho(\alpha)$ or $g_\rho(r, \epsilon, e)$ discussed by Coble (*A class of linear groups with integral coefficients*, Duke Mathematical Journal, vol. 3 (1937), pp. 175–199). (Received May 23, 1938.)

293. E. R. Lorch: *Bicontinuous linear transformations in complex euclidean spaces.*

A complex euclidean space E is a unitary space, a Hilbert space, or a non-separable space of the Hilbert type. A bicontinuous transformation T in E is a bounded linear transformation which possesses a bounded inverse. Let $\{\phi_\alpha\}$ be a complete orthonormal set in E, α ranging over any preassigned set. Let $\{\psi_\alpha\}$ be a set of linearly independent vectors spanning E. (The vectors of a set are said to be linearly independent if, for any n vectors $f_1, \cdots, f_n$ in the set, the condition $\sum_{i=1}^n a_i f_i = 0$ with complex a_i implies $a_i = 0$, $(i = 1, \cdots, n)$.) A complete analysis is given of the internal structure of the set $\{\psi_\alpha\}$ in order that there exist a bicontinuous transformation T such that $T(\phi_\alpha) = \psi_\alpha$. Conditions necessary and sufficient for the existence of T are: (1) the boundedness of the sets of numbers $\{\|\psi_\alpha\|\}$ and $\{\|\psi_\alpha\|^{-1}\}$; (2) the uniform boundedness of the bounds of all projections P constructed as follows: the domain of P is the n-space spanned by any set of n vectors $\{\psi_{\alpha_1}, \cdots, \psi_{\alpha_n}\}$. The range of P is the space spanned by any subset of the set $\{\psi_{\alpha_1}, \cdots, \psi_{\alpha_n}\}$. (Received May 6, 1938.)

294. A. P. Morse: *The behavior of a function on its critical set.*

Consider the statement: If R is an open subset of euclidean space of n dimensions and f is a function on R to the set of real numbers which is of class C^m, $(m \geqq 0, n \geqq 1)$, then f transforms its set of critical points into a set of (linear) measure zero. Whitney (Duke Mathematical Journal, vol. 1 (1935), p. 514) has shown the statement false if $m = n - 1 \geqq 1$; Marston Morse and Sard, in an unpublished paper, have shown on the other hand that the statement is true in case m is the greatest integer in $n + (n-3)^2/16$. The present paper answers in the affirmative the natural question: Is the statement true for $m = n$? This problem was suggested to the author by Marston Morse. (Received May 6, 1938.)

295. B. J. Pettis: *On differentiation in Banach spaces.*

Given an additive function $X(R)$ defined and of bounded variation from euclidean figures to an arbitrary Banach space X, a condition can be stated in terms of linear functionals which is sufficient that $X(R)$ be strongly differentiable a.e. (almost everywhere). From this theorem can be drawn the results of Clarkson (Transactions of this Society, vol. 40 (1936), p. 396), Dunford and Morse (ibid., vol. 40 (1936), p. 415), Gelfond (Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 17), and the author (this Bulletin, vol. 44 (1938), pp. 420–428) concerning differentiation of such functions. The paper also includes proofs of the following theorems: (1) if an additive BV function $X(R)$ is weakly differentiable a.e., then it is strongly differentiable a.e.; and (2) if the space X has an equivalent uniformly convex norm, then X is regular. (Received May 25, 1938.)

296. Olga Taussky: *Differential equations and hypercomplex systems.*

An interesting class of partial differential equations arises in the following way from hypercomplex systems over the field of real numbers. Let S be a hypercomplex system with n base elements $e_1, \cdots, e_n$ and $x_1e_1+\cdots+x_ne_n$ a general element of S, where the x_i are any real numbers. The norm of $x_1e_1+\cdots+x_ne_n$, if defined by means of the regular representation of S, is a homogenous function $f(x_1, \cdots, x_n)$ of the nth degree in $x_1, \cdots, x_n$. If the coordinates are replaced by the differential operators $\partial/\partial x_1, \cdots, \partial/\partial x_n$, a differential operator $f(\partial/\partial x_1, \cdots, \partial/\partial x_n)$ is obtained. When S is the system of complex numbers or quaternions, the differential operators so obtained are the Laplace's operators $\partial^2/\partial x_1^2+\partial^2/\partial x_2^2$ and $\partial^2/\partial x_1^2+\partial^2/\partial x_2^2+\partial^2/\partial x_3^2+\partial^2/\partial x_4^2$, respectively. It is known that each of a pair of functions satisfying the Cauchy-Riemann equations satisfies the Laplace equation; similarly for each of a set of four functions satisfying the Dirac equations. Let $l_i=a_{kj}^{(i)}\,\partial u_k/\partial x_j$, $(i=1, \cdots, n)$, be n linear differential forms with constant coefficients such that $\partial^2u_i/\partial x_1^2+\cdots+\partial^2u_i/\partial x_n^2=a_{i1}\,\partial l_1/\partial x_{i_1}+\cdots+a_{in}\,\partial l_n/\partial x_{i_n}$, $(i=1, \cdots, n)$, where a_{ik} are constants and the i_k are any of the numbers $1, \cdots, n$. The numbers n for which such relations exist can be completely determined by properties of hypercomplex systems. (Received May 27, 1938.)

297. Jesse Douglas: *Green's function and the problem of Plateau.*

Previous solutions given by the author of the general topological form of the problem of Plateau were based on an explicit formula for the Green's function of the basic Riemann surface R in terms of θ-functions and abelian integrals. (See Journal of Mathematics and Physics, vol. 15 (1936), pp. 55–64 and 105–123; also a more comprehensive paper in the Annals of Mathematics (in press), and a note in the Proceedings of the National Academy of Sciences (August, 1938), the last two bearing the title *Minimal surfaces of higher topological structure*). The present paper utilizes Green's function in an intrinsic way, without employing for it any explicit formula. The analytic procedure is the same in all cases, regardless of the particular topological form of R. The central feature is an explicit construction of a special variation of R for the corresponding algebraic curve $\mathcal{A}$, under which Green's function undergoes a particular variation that directly relates the variational condition $\delta A(g, R)=0$ for the fundamental functional $A(g, R)$ to the condition for a minimal surface. A note presenting the main features of the paper will appear in the Proceedings of the National Academy of Sciences (August, 1938), and a detailed version in the Journal of Mathematics and Physics. (Received June 18, 1938.)

298. Jesse Douglas: *Minimal surfaces of higher topological structure.*

This paper first reviews and elaborates the essential features of the author's previous papers on the subject (Journal of Mathematics and Physics, vol. 10 (1931), pp. 310–359; ibid., vol. 15 (1936), pp. 55–64, 105–123; Transactions of this Society, vol. 33 (1931), pp. 263–321; ibid., vol. 34 (1932), pp. 731–756). It then provides various new results, relating particularly to the case where the prescribed genus of the minimal surface is greater than zero. An explicit formula in terms of θ-functions and abelian integrals is given for the Green's function of a general Riemann surface R, with any finite number of boundaries and of any finite genus. As was announced in the last cited Journal publication (p. 108), the theory is extended to the case where the contours are completely general Jordan curves, not bounding any surface of finite area. As throughout the author's work on the Plateau problem, the conformal mapping of plane regions is included. A more detailed discussion is given of a certain inequality necessary for the application of the general theory to this particular case. A comprehensive presentation will appear in the Annals of Mathematics and the Proceedings of the National Academy of Sciences (August, 1938). (Received June 16, 1938.)

299. N. A. Hall: *The number of representations function for binary quadratic forms.*

Explicit expressions are given for the number of representations of any positive integer in any positive definite binary quadratic form of discriminant such that there is a single class in each genus. To apply the Dirichlet formula for the number of representations for integers prime to the discriminant, the reduction formulas of G. Pall (Mathematische Zeitschrift, vol. 36 (1932), pp. 321–343) for the representations of integers not prime to the discriminant are extended. These show that aside from a few trivial exceptions specially treated, when the discriminant contains no odd prime square factors, the occurrence of factors of the discriminant in the number to be represented affects only the representations or non-representation; and in case it is represented, does not affect the number of representations. The representability is a function of the power of the prime factors of the discriminant and the characters for the particular form involved. This is given explicitly according to a convenient congruential classification of discriminants. When the discriminant contains odd prime square factors, use is made of theorems previously obtained by the author (cf. this Bulletin, vol. 43 (1937), p. 202) showing that only ten such cases occur when there is a single class of forms to a genus. This permits treatment of these cases individually and thus completes the solution. (Received June 14, 1938.)

300. E. R. Kolchin: *On complementary manifolds in certain Banach spaces.*

Let $\mathfrak{B}$ be a Banach space. Consider the question "Does every closed linear manifold in $\mathfrak{B}$ have a complementary manifold?" F. J. Murray has shown (Transactions of this Society, vol. 41 (1937), pp. 138–152) that the answer for $\mathfrak{B}=L_p$ or l_p, $(1<p\neq 2)$, is "no." In this note the arguments of Murray are modified to show that the answer for $\mathfrak{B}=c$, c_0, or m is "no." (Received June 3, 1938.)

301. Walter Leighton: *Sufficient conditions for the convergence of a continued fraction.*

Let (1) $1+K_1^\infty[a_n/1]$ be a continued fraction in which the quantities a_n are arbitrary complex numbers. Sets of sufficient conditions for convergence of (1), established by J. Worpitsky, E. B. Van Vleck, A. Pringsheim, O. Szász, H. S. Wall, W. Leighton, and others, have required that at least an infinite subsequence of the numbers a_n be less than 1/4 in absolute value. This condition can be removed as follows. If $|1+a_2|\geqq 1+|a_1|$, $|a_{2n+1}|\leqq m$, $|a_{2n}|\geqq(2+m)/(1-m)$, $|a_{2n+2}|\geqq 2+m+m|a_{2n}|$, $n=1, 2, 3, \cdots$, where m is any fixed positive number <1, the continued fraction (1) converges. A similar theorem with the roles of the "odd" and "even" a_n interchanged is also valid. In particular, the continued fraction with $a_n=-1/4-e_n$ can be shown to converge for a wide choice of the numbers $e_n>0$, even though if $e_1=e_2=e_3=\cdots=e$, (1) diverges (Szász) for all $e>0$. (Received June 4, 1938.)

302. B. A. Lengyel: *Bounded self-adjoint operators and the problem of moments.*

In this paper, a connection is established between the theory of moments and the theory of bounded self-adjoint operators. Let H be a bounded self-adjoint operator of the Hilbert space. The resolvent, R_z of H, can be represented by a Stieltjes integral with the denominator $z-\lambda$, if the problem of moments with $\mu_n=(H^nf, f)$ has a unique solution for all f's in the Hilbert space. The existence of a solution is immediately secured by the definite positive character of the quadratic forms $|\sum_{i=0}^{n}x_iH^if|^2=\sum_{i=0}^{n}x_i\bar{x}_j(H^if, H^jf)$, $(n=1, 2, \cdots)$. The uniqueness of the solution is established through a lemma concerning the closed linear manifolds generated by $[Hf, H^2f, H^3f, \cdots]$. Since it is known that the spectral theorem follows readily from the Stieltjes integral representation of the resolvent, this paper can be considered as one of the numerous proofs of the spectral theorem. (Received June 20, 1938.)

303. Deane Montgomery and Leo Zippin: *Non-abelian compact connected transformation groups of three-space.*

In this paper, the authors conclude their study of *compact connected effective transformation groups of euclidean three-space* (hereafter, $Tg(E_3)$) with a final theory on the non-abelian case. The following result, already established by the authors, is used: The only *abelian* $Tg(E_3)$ is, abstractly, the group of rotations of a circle. Moreover, if K denotes such a group, the space E_3 must admit a coordinate system in which K is the group of all rigid rotations of E_3 about a fixed axis. This is now complemented by the following theorem: The only *non-abelian* $Tg(E_3)$ is, abstractly, the group of rotations of the two-sphere. Moreover, if G' denotes such a group, the space E_3 must admit a coordinate system in which G' is the group of all rigid rotations of E_3 about the origin. (Received June 16, 1938.)

304. A. R. Schweitzer: *An outline of the history and the philosophy of the concept of orientation.* Preliminary report.

Fundamental concepts of this paper are Reason and the Hand. The author analyzes critically various interpretations of orientation in ancient and modern literature and attempts to coordinate them in a unified whole with reference to their mathematical implications. The subject matter falls roughly into three categories: I, mystical speculation and theology; II_1, philosophy, metaphysics, psychology, and logic; II_2, aesthetics, art; III, science, mathematics. The corresponding main sources are: I, the scriptures and the works of Galen; II_1, Kant, early treatises and treatise on subjective orientation (1786), "Gestalt" psychologies of von Ehrenfels, Koehler, and Koffka; II_2, Leonardo da Vinci; III, Fresnel and Pasteur in science and Moebius, Grassmann, Klein, and Study in mathematics. Among the interpretations of orientation considered, are: right and left, mirror image, symmetry, balance; position, order; pointing, direction, polarity; motion, twist; inversion, asymmetry, and so on. The principal subjects considered are: in science, crystallography and cytology; physics, chemistry; biology, pathology; and in mathematics, geometry and topology. Orientation as such has been studied by G. Reynaud (1898), T. A. Cook (1914), F. M. Jaeger (1920), G. W. Crile (1926), W. Ludwig (1932), C. W. R. Hooker (1934), and others. In connection with subjective orientation, the author refers to his article in Revue de Metaphysique et de Morale, vol. 22 (1914). (Received June 11, 1938.)

305. A. H. Taub: *Spin representation of conformal groups.*

The conformal group of a flat n-space is isomorphic with the orthogonal group of a flat $(n+2)$-space. By using the spinor representation of the latter, a spin representation of the conformal group may be obtained. Also, a consideration of the form of the first isomorphism in detail readily yields, in the $(n+2)$-space, the images of rotations, translations, inversions, and dilations in the n-space. These make it possible to find the explicit forms of the spin transformations corresponding to each of the different types of transformations in the n-space. (Received June 18, 1938.)

306. A. H. Taub: *Tensor equations equivalent to the Dirac equations.*

When the identities satisfied by the matrices which occur in the Dirac equation are used, the tensor equations equivalent to the Dirac equation obtained by E. T. Whittaker (*On the relations of the tensor-calculus to the spinor-calculus*, Proceedings of the Royal Society, vol. 158 (1937), pp. 38–46) follow immediately. Another set of tensor equations is also obtained. These equations contain the current vector, and the divergence of an anti-symmetric tensor which describes the magnetic moment of the electron. It is also shown that the Dirac equations can be written in an equivalent spinor form. This form is used to calculate the "acceleration" in the Dirac theory, and it is shown to be equal to the divergence of a symmetric second order tensor which is similar to the stress energy tensor of the Maxwell theory. (Received June 18, 1938.)

307. W. L. Williams: *Permanent configurations in the problem of five bodies.*

A permanent configuration is a configuration of n bodies which has the property that the ratio of distances between corresponding bodies is constant. In other words, the figure may change in size, but not in shape. In the present paper, necessary and sufficient conditions for any plane configuration of five bodies (other than that in which all are in a straight line) are given, and also a detailed analysis of the various types of such configurations, both convex and concave, that can be formed by rearranging the bodies to give pentagons of different shapes. Several numerical examples are worked out, including the regular pentagon with one mass at each vertex and the square with one mass at each corner and the fifth mass at its center. (Received June 10, 1938.)

308. Reinhold Baer: *The significance of the system of subgroups for the structure of the group.*

If G and H are isomorphic groups, then their sets of subgroups are isomorphic and every isomorphism of G upon H induces an isomorphism of the set of subgroups of G upon the set of subgroups of H. If, conversely, G is an abelian group and G and H have isomorphic sets of subgroups, then G and H are isomorphic groups, provided the correspondence between the sets of subgroups preserves normality and indices. If G is a primary hamiltonian group of "not too small" an abelian group, then it is even possible to prove that every isomorphism of the set of subgroups of G is induced by an isomorphism of the group G. (Received June 27, 1938.)

309. E. F. Beckenbach and Maxwell Reade: *A characterization of minimal surfaces.*

Continuing the work reported in abstract 44-3-92, the authors prove the following theorem: If $x_j(u, v)$, $(j=1, 2, 3)$, are of class C_3 (that is, all derivatives of order less than or equal to three are continuous) in a domain D, then a necessary and sufficient condition that these functions be the coordinate functions of a minimal surface given in (harmonic and) isothermic representation is that for each circle C lying in and enclosing only points of D, $\sum_{j=1}^{3}[\int_C x_j(u, v)(du+idv)]^2=0$. (Received June 24, 1938.)

310. Archie Blake: *Criteria for the reality of apparent periodicities and other regularities*. Preliminary report.

The following procedure is suggested to provide a more efficient statistic than the classical periodogram in the case of phenomena which are not nearly sinusoidal in type. Denote the observed function (after a suitable change of scale, if necessary) by $f(t)$, and let there be given also a weight function $w(t)$ and a period T to be tested. Compute the departure $d(t)$ by referring f to its mean. As a measure of the covariance, or contribution of the observations at any two times t_1 and t_2 to the periodicity, adopt a function $C(d(t_1), d(t_2); w(t_1), w(t_2); t_1, t_2; T)$, and define the periodicity $P(T)$ as the double integral of C with respect to t_1 and t_2. The function C is to be chosen to suit the needs of the type of problem in hand; there is considerable latitude here, by which such phenomena as quasi-persistence, seismic aftershocks, and line spectra can be appropriately treated. For the detection of certain other preassigned types of regularity than pure periodicity, the method of analysis is essentially the same, the parameters under investigation being used as arguments of C in place of T. (Received June 21, 1938.)

311. Jesse Douglas: *The most general form of the problem of Plateau.*

Suppose there is given any riemannian manifold R in the most general sense of the term, that is, any two-dimensional connected topological variety for which there is defined in the neighborhood of each point a local conformal representation on a circle. The manifold R may then have any finite or infinite number of boundaries, and any topological structure whatever, that is, any finite or infinite type of connectivity. It may also have either character of orientability, that is, one- or two-sidedness. Consider also any point set Γ in n-dimensional euclidean space which is a topological image of the total boundary C of R; the point set Γ may consist of any finite or infinite number of Jordan curves, together with their limit points; or it may be some more general type of point set. A definite sense of description is associated with each component of Γ, carried over from C, which may be supposed to be oriented so that R is on the left. The problem is to determine the existence of a minimal surface M topologically equivalent to R and bounded by Γ. This problem is completely solved in the present paper. A note will be published in the Proceedings of the National Academy of Sciences, and a more detailed presentation in the Journal of Mathematics and Physics. (Received June 28, 1938.)

312. Jesse Douglas: *Remarks on Riemann's doctoral dissertation.*

This paper will appear in full in the Proceedings of the National Academy of Sciences, July, 1938. (Received June 28, 1938.)

313. Aaron Fialkow: *Totally geodesic Einstein spaces.*

In this note, characteristic conditions under which an Einstein space E_m admits families of totally geodesic Einstein subspaces E_n are obtained. The results for the case $m>n+1$ have been stated by the author (cf. Proceedings of the National Academy of Sciences, vol. 24 (1938)) and are related to the problem of imbedding an E_m as a hypersurface of a space of constant curvature. If $m=n+1$, theorems such as the following are obtained: A one-parameter family of isometric E_n's can be imbedded as ∞^1 non-parallel totally geodesic hypersurfaces of an E_{n+1} if and only if each E_n can be mapped conformally on another Einstein space by means of a function σ with $\Delta_1\sigma \neq 0$. If a and b are the mean curvatures of E_{n+1} and E_n, respectively, then $nb = (n-1)a$. (Received June 28, 1938.)

314. O. H. Hamilton: *Some theorems concerning points and continua left invariant by transformations of continua not locally connected.*

The following results are obtained. If M is a compact continuum in the plane S which does not separate S, then every reversibly continuous transformation of M into a subset of itself leaves invariant some point, some continuum which is the sum of a simply connected domain and its boundary, or some continuum whose boundary is indecomposable. If M, in addition to satisfying the requirements listed above, does not contain an indecomposable continuum in its boundary, then every reversibly continuous transformation of M into itself leaves some point invariant. Finally, these theorems, with modifications, are generalized for continua in a metric space. (Received June 23, 1938.)

315. Archibald Henderson and J. W. Lasley: *On harmonic separation.*

The authors present an historical account of the development of harmonic separation from the time of the ancient Greeks to the present. The steps in the evolution of the linear construction for harmonic conjugates are outlined from Serenus to Chasles. Other harmonic constructions are given; some involving two circles, some one circle. The notions of inversion and of similar triangles are employed. Analytic methods applied to the bisector problem and to the circle of Apollonius yield expressions for the foregoing geometric solutions in algebraic form. (Received June 27, 1938.)

316. E. V. Huntington: *Note on a recent set of postulates for the calculus of propositions.*

In my paper, *Postulates for assertion, conjunction, negation, and equality* (Proceedings of the American Academy of Arts and Sciences, vol. 72 (1937), pp. 1–44), in which the postulates 1–17 were expressed, after Lewis, in terms of the base K, T, $\times$, $'$, $\equiv$, the definition $a \prec b : = : ab' \equiv Z$ may be replaced by the simpler form $a \prec b : = : ab \equiv a$ provided the theorem $a \prec b . = . b' \prec a'$ is added to the list of postulates. In the revised list thus obtained, the three special elements Z, Z', Z^* appear only by inference, no explicit mention of them in the postulates being required. (Received June 25, 1938.)

317. N. H. McCoy: *Generalized regular rings.*

Let $\mathfrak{R}$ be a ring with unit element. If for every element a of $\mathfrak{R}$ there exists a positive integer n such that a^n is regular (von Neumann, Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 707–713, it is said that $\mathfrak{R}$ is *π-regular*. If there is a fixed integer m such that for all a in $\mathfrak{R}$, a^m is regular, $\mathfrak{R}$ is said to be *m-regular*. The purpose of the present note is to prove a few theorems concerning π-regular and m-regular rings. For the most part, they are analogs of known theorems on regular rings. (Received June 23, 1938.)

318. E. W. Paxson: *Induced norms in topological groups.*

It is shown that, in the presence of connectedness, an extension of the concept of convexity, and sequential completeness, the notions of linearity and norm as ideas relative to a designated neighborhood of the identity may be induced in a space that supports simply a Hausdorff group structure. In a sense, these results supplement those of J. von Neumann (Transactions of this Society, vol. 37 (1935), pp. 1–20) on pseudo-metrics in linear topological spaces. (Received June 27, 1938.)

319. H. S. Carslaw and J. C. Jaeger: *On Green's functions in the theory of heat conduction.*

In this note, the second of the problems discussed by Lowan in his paper *On the operational determination of two dimensional Green's functions in the theory of heat conduction* (this Bulletin, vol. 44 (1938), pp. 125–133) is solved, first, by contour integration and, second, by the use of the Laplace transformation. The connection between the two methods is pointed out and an error in Lowan's result is corrected. (Received June 25, 1938.)

320. W. H. Gage: *Class number relations for the form* x^2-2y^2.

By means of the expansions for $\phi(x, y, z)$ (American Journal of Mathematics, vol. 59 (1937), pp. 919–920), and by the method of paraphrase, a number of the class number relations of Petr and Humbert have been obtained. These relations involve sums of ideal divisors of an integer $n=x^2-2y^2$. A number of new relations for $n-u^2=x^2-2y^2$, $n-u=x^2-2y^2$, n fixed and u variable, are also given. All relations are obtained by equating coefficients of q^n in identities involving theta expansions. Class numbers are introduced in the identities through the formulas for the numbers of representations of integers in the forms $xy+yz+zx$, $xy+yz+2zx$, $xy+2yz+2zx$. (Received July 1, 1938.)

321. Henry Gerhardt: *The generalization of Miquel's theorem.*

The purpose of this paper is to determine whether Miquel's theorem is valid in hyperbolic geometry. A conformal representation of the hyperbolic plane on the euclidean plane is used, with straight lines and circles in the former represented by half-circles and circles in the latter. It is found that Miquel's theorem is not valid in hyperbolic geometry, but a generalized form of the theorem is obtained which is valid in both hyperbolic and euclidean geometry. This method of generalizing theorems of euclidean geometry by conformal representation of the hyperbolic plane seems to be a fertile one. (Received June 25, 1938.)

322. J. L. Walsh and W. E. Sewell: *Note on degree of trigonometric and polynomial approximation to an analytic function.*

In this note the authors assume $f(z)$ to be analytic in the annulus $\rho>|z|>1/\rho<1$ and continuous in the corresponding closed region, and show that certain continuity properties (Lipschitz conditions on function or derivatives) of $f(z)$ on $|z|=\rho$ and $|z|=1/\rho$ imply a specific degree of convergence of various trigonometric polynomials and polynomials in z and $1/z$ to $f(z)$ on $|z|=1$. Conversely, it is shown that a given degree of convergence on $|z|=1$ implies certain continuity properties of $f(z)$ on $|z|=\rho$ and $|z|=1/\rho$. By well known transformations, these results are extended to a function $f(z)$ analytic in an ellipse whose foci are $+1$ and -1 and the semi-sum of whose axes is ρ, and to a function $f(z)$ periodic with period 2π and analytic in the band $|y|<\log\rho$, where $z=x+iy$. (Received July 1, 1938.)