ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

259. E. E. Betz: *On accessibility and separation by simple closed curves.*

In this note it is shown that if M is a Peano continuum such that for every simple closed curve J in M the set $M-J$ consists of at least two and at most a finite number of components, then every boundary point of each such component D is regularly accessible from D. This theorem leads to new characterizations of the simple closed surface. (Received April 6, 1939.)

260. Garrett Birkhoff: *Lattice theory of Brouwerian logic.*

Brouwerian logics can be defined as lattices with 0 and I and an implication operation $x\rightarrow y$ which satisfies (1) $x\rightarrow(y\rightarrow z)=(x \cup y)\rightarrow z$, (2) $x\rightarrow y=0$ if and only if $x\geqq y$. It can then be proved that no lattice will support more than one implication operation making it into a Brouwerian logic, and that a lattice will support as many as one such implication operation if and only if it satisfies a certan non-self-dual infinite distributive law. (Received April 10, 1939.)

261. R. P. Boas (National Research Fellow): *Integral functions bounded on the real axis.*

Let $f(z)$ be an integral function of order one and type $k<\pi$. If the numbers $f(\lambda_n)$ are bounded, where the λ_n are real, and $|\lambda_n-n|\leqq 1/(2\pi^2)$, $(n=0, \pm 1, \pm 2, \cdots)$, then $f(x)$ is bounded for real x. The proof involves the Paley-Wiener theory of non-harmonic Fourier series. When $\lambda_n=n$, the theorem reduces to a theorem of Miss Cartwright, for which a new proof is incidentally obtained. (Received April 15, 1939.)

262. R. H. Cameron: *Extensions of Wiener's general Tauberian theorem.*

If $f(x)$ is bounded and measurable and its faltung with each function $g(x)$ of L approaches $A\int g(x)dx$ as $x\rightarrow\infty$, we say that $\text{wlim}\, f(x)=A$, the limits of all integrals being $-\infty$ to ∞. Then one has the theorems: (1) Let $g(x)$ be a function of bounded variation such that $\int e(tx)dg(x)$ never vanishes. Then if $f(x)$ is bounded, Lebesgue measurable, and Radon measurable with respect to $g(t-\lambda)$ for all λ, and if $\int f(x-t)dg(t)\rightarrow A\int dg(t)$ as $x\rightarrow\infty$, it follows that $\text{wlim}\, f(x)=A$. (2) Let $g(x)$ be a function of bounded variation whose discrete and singular parts are $h(x)$ and $s(x)$; and let lower bound $|\int e(tx)dg(t)|>0$ and lower bound $|\int e(tx)dh(t)|>\int|ds(t)|$. Then if $f(x)$ is bounded and Radon measurable, $\int f(x-t)ds(t)$ is a Baire function, and $\int f(x-t)dg(t)\rightarrow A\int dg(t)$, then it follows that $\lim f(x)=A$. In this case the limit is an ordinary (strong) limit. (Received April 20, 1939.)

263. C. R. Cassity: *Two theorems on the rational quartic surface in space of four dimensions.* Preliminary report.

The first of these theorems gives explicit equations for the cones in the pencil of quadrics on the quartic surface in terms of the coordinates of the fundamental points of the plane representation. These equations are such that all members of the pencil are written as sums of squares. The second theorem relates the elliptic and hyperbolic points of the surface as defined by Fabricius-Bjerre (Mathematische Zeitschrift, vol. 41 (1936), pp. 686–707) to the map on the surface as studied by the author in an article soon to appear in the Duke Mathematical Journal. (Received April 1, 1939.)

264. J. J. DeCicco: *The differential geometry of the curves of the Kasner plane.*

This is a continuation of two papers by Kasner (Science, vol. 85 (1937), pp. 480–482; Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 337–341), abstract 42-11-398 by Kasner, and abstracts 44-11-444 and 45-1-83 by the author. This paper studies the differential geometry of the curves of the Kasner plane K_2 with respect to the fundamental three-parameter group G_3, which is induced by the group of direct conformal transformations. The limit R of the ratio of the arc of any curve to its chord is found to consist of all rational numbers R such that R^2-R is the square of a rational number. If at every point of a curve the number R is unity, the curve is said to be a general curve. For a general curve $y=y(x)$, the length of arc is given by $s=\int y'^{-1}dx$ and its curvature K is given by $y''/2$. The intrinsic equation of a general curve is $K=K(s)$ where K is a function of s. We find the necessary and sufficient condition that ∞^1 parabolic-circles be a set of osculating parabolic-circles. The theory of evolutes and involutes is developed. Finally the differential geometry of fields of lineal elements of this plane is briefly considered. (Received April 24, 1939.)

265. J. J. DeCicco: *The conics of the Kasner plane.*

This paper studies the geometry of the conics of the Kasner plane with respect to the Kasner group G_3. The locus of all points of this plane which satisfy a quadratic equation is called a conic. The conics may be classified into twelve distinct types with respect to the group G_3. The number of invariants of each type is 0, 1, 2. A geometric construction for the conics is found and a geometric interpretation for each of the invariants. A general conic is the locus of a point such that the ratio $\epsilon\neq 0$ of its distance from a fixed point (the focus) to its distance from a fixed general line (the directrix) is constant. The general conic is an ellipse, parabola, or hyperbola according as $0<\epsilon<1$, $\epsilon=1$, or $\epsilon>1$ or $\epsilon<0$. A general central conic has two foci which lie on the conic and two parallel directrices. The minor axis of a general conic is orthogonal (dihorn angle 1/2) to the directrices. Finally a general central conic is the locus of a point such that the sum of its distances from two fixed points (the foci) is equal to a nonzero constant. (Received April 24, 1939.)

266. R. P. Dilworth: *A characterization of complemented modular lattices.*

A lattice Σ is said to be (1) an exchange lattice of type II if $(a, b)>a$ implies $b>[a, b]$, (2) a Jordan lattice if all maximal chains joining two given elements have the same length, (3) relatively complemented if $a\supset b$ implies there exists b' such that $(b, b')=a$, $[b, b']=z$ where z is the null element of Σ. K. Husimi has conjectured that a relatively complemented lattice Σ is modular if every relatively complemented sublattice is a Jordan lattice. It is shown here that the theorem in the above form does not hold but that a lattice satisfying the hypotheses of the theorem is an exchange lattice of type II. Hence if the dual hypothesis is added to the theorem, the conclusion that Σ is modular is correct. The following theorem is also proved: Every archimedean complemented, non-modular lattice with unit element i and null element z has a sublattice $\{i, a, b, c, z\}$ where $a \supset c$, $(b, c)=i$ and $[a, b]=z$. As an immediate corollary we have the theorem due to G. Birkhoff and M. Ward (abstract 39-1-78) that a lattice of finite dimensions is a Boolean algebra if and only if every element of the lattice has a unique complement. (Received May 1, 1939.)

267. R. P. Dilworth: *Note on the prime elements of a modular lattice.*

An element p of modular lattice is said to be prime if $p \supset [a, b]$ implies $p \supset a$ or $p \supset b$. An irreducible q is said to be isolated if when it occurs in the reduced representation of an element as a crosscut of irreducibles it occurs in every such representation. It is then shown that an element p of a modular lattice in which the ascending chain condition holds is a prime if and only if it is an isolated irreducible. It is also shown that a lattice with chain condition is distributive if and only if every irreducible is a prime. As an application of these two theorems we have the result due to G. Birkhoff that a modular lattice in which every element has a unique representation as a reduced crosscut of irreducibles is distributive. (Received May 1, 1939.)

268. D. W. Hall (National Research Fellow): *On new characterizations of the 2-sphere.* Preliminary report.

In this note an accessibility theorem of E. E. Betz (see abstract 45-7-259) is employed to demonstrate the following theorem: If M is a compact continuum separated by no pair of its points but such that there exists an integer N such that for every simple closed curve J in M the set $M-J$ has at least two and at most N components, then M is a 2-dimensional sphere. The case $N=2$ gives a known theorem of Leo Zippin. Other characterizations are also obtained. (Received April 7, 1939.)

269. M. R. Hestenes: *On the first necessary condition for minima of double integrals.*

For a simple double integral problem in the calculus of variations the first variation of the double integral is of the form $L(\eta)=\iint_A(u\eta_x+v\eta_y+w\eta)dxdy$. It is assumed that A is a region in xy-space whose boundary C is composed of a finite number of disjoint simply closed regular arcs and that $A+C$ can be divided into a finite number of parts bounded by simply closed regular arcs on each of which the functions u, v, w are continuous. In the present paper it is shown that if $L(\eta)=0$ for every admissible variation η having $\eta\equiv 0$ on C, then $L(\eta)=\int_C\eta(udy-vdx)$ for every admissible variation η. Setting $\eta\equiv 1$ one obtains the Coral-Haar equations. The proof is direct and does not involve the use of the fundamental lemma for simple or double integrals. Further consequences of this result are given. (Received April 14, 1939.)

270. E. R. Lorch: *Means of iterated transformations in reflexive vector spaces.*

The paper gives a proof of the following "mean ergodic theorem": Let V be a bounded linear transformation in a reflexive vector space $\mathfrak{B}$ whose iterates V^n, $n=0, 1, 2, \cdots$, are uniformly bounded, $|V^n|\leqq K$. Then the means $1/n\sum_{s=0}^{n-1}V^s$ converge strongly to a limiting transformation which is a projection P $(P^2=P)$, with $|P| \leqq K$. $Pf=f$ if and only if $Vf=f$. $Pf=0$ if and only if f is in the closure of the range of $V-I$ $(I=V^0)$. The theorem includes as special cases the results of F. Riesz as well as their recent generalization (for the case $K \leqq 1$) by Garrett Birkhoff. The following theorem is decisive in the proof: Let $\mathfrak{B}$ be reflexive, T a bounded linear transformation in $\mathfrak{B}$. Let $\mathfrak{M} \subset \mathfrak{B}$ be the manifold on which $Tf=0$, $\mathfrak{N}$ be the closure of the range of T. Let $\bar{T}$ be the adjoint of T defined over $(\mathfrak{B})$, the adjoint space of $\mathfrak{B}$; let $(\mathfrak{M})$ and $(\mathfrak{N})$ be defined for $\bar{T}$ as indicated. Then $(\mathfrak{M}) = \mathfrak{N}^{\perp}$, the orthogonal complement of $\mathfrak{N}$, $(\mathfrak{N}) = \mathfrak{M}^{\perp}$, $\mathfrak{M} = (\mathfrak{N})^{\perp}$, and $\mathfrak{N} = (\mathfrak{M})^{\perp}$. (Received April 10, 1939.)

271. A. D. Michal and A. B. Mewborn: *General flat projective geometry.*

In this study the authors first generalize the notion of a flat projective space of Veblen and Whitehead (Cambridge Tracts in Mathematics and Mathematical Physics, no. 29, 1932) by replacing their arithmetic space of projective coordinates with a Banach space B_1 of couples (x, x^0) as projective coordinates, where x is in a Banach space B of allowable coordinates and x^0 is a real variable. It is then shown that any transformation of coordinates from allowable coordinates in B to general projective coordinates in B_1 satisfies a completely integrable abstract second order differential equation. A new existence theorem for a system of completely integrable abstract first order differential equations is then proved. This theorem is applied to the characterization of a flat projective geometry as a subclass of general curved projective geometry treated in the previous paper of the authors (*General projective differential geometry of paths*). (Received April 15, 1939.)

272. A. N. Milgram: *Partially ordered sets: bounds and mappings.*

We consider a partially ordered set A and a lowering function $l(a) \leqq a$ defined on A. Sufficient conditions are given in order that there exist an element a such that $l(a)=a$. The subset U of A is called an upper section if $a<b$ and $a \in U$ imply $b \in U$. A system of upper sections σ with the property that $a<b$ implies there is an element $U \in \sigma$ such that $a \notin U$ and $b \in U$ is called a separating system of A. If each well-ordered decreasing sequence of power at most P has a lower bound in A, and if A has a separating system of power P, then there is always an element a such that $l(a)=a$. This depends on the fact that no well-ordered decreasing sequence of elements of A can have a power greater than P. In the second section a type of universal ordered set is given into which all partially ordered sets may be mapped so that $a \neq b$ implies $f(a) \neq f(b)$ and $a<b$ implies $f(a)<f(b)$. The universal set is a generalization of the Cantor discontinuum to the transfinite. (Received April 10, 1939.)

273. Oystein Ore and J. E. Eaton: *Remarks on multigroups.*

This paper contains various contributions which lead to simplifications and improvements in certain parts of the previous theory of multigroups. (Received April 25, 1939.)

274. J. F. Ritt: *On intersections of algebraic differential manifolds.*

An example is presented which exhibits a fundamental difference between the dimensionality theory of algebraic manifolds and that of differential manifolds. In this example, three unknown functions are uniquely determined by what may quite fairly be called two conditions. (Received April 5, 1939.)

275. J. F. Ritt and E. R. Kolchin: *On certain ideals of differential polynomials.*

This note deals with the decomposition theory of ideals whose manifolds are composed of several disjoint submanifolds. An examination is made of the case in which one of the submanifolds consists of a single solution. (Received April 5, 1939.)

276. L. D. Rodabaugh: *On the partial differential equation* $\partial z/\partial x + f(x,y)\partial z/\partial y = 0$.

Theorem 1. Assume that g is a bounded, open, simply connected plane region and $f(x, y)$ is a function such that (a) $f(x, y)$ and $f_y(x, y)$ are defined and are continuous in g; (b) $f(x, y)$ and $f_y(x, y)$ have definite finite continuous limits on the boundary of g. Assume also that L and U are any finite real numbers such that L is less than U. Then there exists a function $I(x, y)$ such that, in g: (a) $I(x, y)$ is defined and is of class C' with respect to x and y; (b) $I(x, y)$ satisfies the partial differential equation $\partial z/\partial x + f(x, y)\partial z/\partial y = 0$; (c) $I_y(x, y) > 0$; (d) $L < I(x, y) < U$. Theorem 2 generalizes Theorem 1 to the case of a doubly connected region; extension is also made to the case of a multiply connected region of finite order of connectivity greater than two. The paper deals only with real functions of real variables. (Received April 1, 1939.)

277. W. T. Scott and H. S. Wall: *A convergence theorem for continued fractions.*

The continued fraction $[a_n/1]_1^\infty$, in which the a_n are complex numbers, converges if there exists a convergent series of positive numbers $p_1 + p_2 + p_3 + \cdots$ such that $r_1|1+a_2| \geqq |a_2|$, $r_2|1+a_2+a_3| \geqq |a_3|$; $r_n|1+a_n+a_{n+1}| \geqq r_n r_{n-2}|a_n| + |a_{n+1}|$, $(n=3, 4, 5, \cdots)$, where $r_n = p_{n+1}/p_n$, $(n=1, 2, 3, \cdots)$. The power of this theorem is illustrated by the following result which is obtained from it: The continued fraction $[a_n/1]_1^\infty$ converges if the a_n are contained in any closed bounded region lying within the parabola $|z| - R(z) = 1/2$. That this is the best such theorem about regions containing the origin and symmetric with respect to the real axis is easily seen since the continued fraction $a/1 + \bar{a}/1 + a/1 + \bar{a}/1 + \cdots$ diverges when a is outside the parabola. (Received May 12, 1939.)

278. M. F. Smiley: *A note on measure functions in a lattice.*

This note presents a generalization of Carathéodory's criterion of measurability which is applicable to an arbitrary lattice. It is shown that for modular lattices the "measurable" elements form a sublattice. The question of closure of this sublattice under denumerable sums and products is discussed. Some of the properties of regular real valued lattice functions (cf. Carathéodory, *Vorlesungen über reelle Funktionen*, 2d edition, p. 258) are derived. (Received May 1, 1939.)

279. A. D. Wallace: *An axiomatic treatment of separation.* Preliminary report.

In a space S, take as undefined the notion of two point sets being mutually separated. To indicate this relation, write $X|Y$. The following conditions are assumed: (1) $X|Y$ implies $Y|X$; (2) $X|Y$ implies $X \neq 0 \neq Y$; (3) $0 \neq X \subset Y$ and $Y|Z$ imply $X|Z$; (4) $X|Y$ and $X|Z$ imply $X|(Y+Z)$; (5) $X|Y$ implies $XY = 0$. A set is said to be connected provided that it is not the sum of two mutually separated sets.

On the basis of the above axioms, it does not follow that two distinct points are mutually separated. Nevertheless a large body of theorems concerning connected sets holds true. For example, the following result is obtained: If X is a connected subset of the connected set Y and $Y-X=Y_1+Y_2$ where $Y_1 | Y_2$, then Y_1+X and Y_2+X are connected; and if Z is a component of $Y-X$, then $Y-Z$ is connected. There are various ways in which a topology may be introduced into S. For a "closure" topology, define the closure of the set X to be the set of all points not separated from X. It is also possible to introduce a "neighborhood" topology in several different ways. The space S may be considered as a "discrete space" (Linfield) or as a space of "contiguous points" (R. L. Moore). (Received April 6, 1939.)

280. Morgan Ward: *An arithmetical characterization of a modular lattice.*

Let $\mathfrak{S}$ be a lattice with respect to a division relation $x \supset y$, and let $[x, y]$ denote the crosscut of x and y. An element q of $\mathfrak{S}$ is said to be irreducible if $q=[v, w]$ implies $q=v$ or $q=w$. An irreducible is said to be a component of an element u of $\mathfrak{S}$ if $u=[q, v]$ and $q \not\supset v$. If q is a component of u, any element $\bar{q}$ such that $u=[q, \bar{q}]$, $q \not\supset \bar{q}$ is called a complement of q in u. It is proved in this paper that if the ascending chain condition holds in $\mathfrak{S}$, then the following condition is both necessary and sufficient that $\mathfrak{S}$ be a modular lattice: If q is irreducible, and c any element such that neither $q \supset c$ nor $c \supset q$, then q is a component of every element d of the quotient lattice $q/[q, c]$; and if $d \neq q$, q has at least one complement in d dividing c. (Received April 3, 1939.)

281. G. T. Whyburn: *Non-alternating interior retracting transformations.*

Let M be a compact locally connected continuum, let axb be any simple arc in M, and let J be any simple closed curve in M. It is shown that there exists a non-alternating transformation which retracts M into axb and is interior on the cyclic chain $C(a, b)$. Similarly, if M is not unicoherent about J, there exists a non-alternating transformation which retracts M into J and is interior on the cyclic element $C(J)$ of M containing J. Also it is shown that M will be mappable onto a circle by a non-alternating interior transformation if and only if M is cyclic and non-unicoherent. (Received April 12, 1939.)

282. L. E. Mehlenbacher: *Determination of the asymptotic behavior of the solutions of differential equations of the Fuchsian type; the case of $n+2$ regular singular points.*

This paper deals with the linear homogeneous differential equation of the second order $z^2[a_\alpha z^\alpha]d^2y/dz^2+z[b_\alpha z^\alpha]dy/dz+c_\alpha z^\alpha y=0$, where α denotes summation from 1 to n. According to the Fuchsian theory, this equation has $n+2$ regular singular points, $z=0$, $z=\infty$, and the n zeros $z=z_i$, $(i=1, 2, 3, \cdots, n)$, of $a_\alpha z^\alpha=0$, at each of which there exist two solutions each expressible in an infinite series. The solutions at $z=0$ are denoted by y_1, y_2 and those about $z=\infty$ by $\bar{y}_1$, $\bar{y}_2$. The problem of the paper is to determine the precise manner in which each of the solutions y_1, y_2 is connected linearly with the two solutions $\bar{y}_1$, $\bar{y}_2$; similar relations for the solutions about the points $z=z_i$ being obtained by similar methods. Methods used are based upon the work of Nörlund in difference equations. The results generalize those of W. B. Ford, who treated the case in which $n=2$. (Received May 13, 1939.)

283. Barkley Rosser: *A new lower bound for the exponent in the first case of Fermat's last theorem.*

This paper proves the theorem: If p is an odd prime and $a^p+b^p+c^p=0$ has a solution in integers prime to p, then $p>41,000,000$. It seems certain that still higher lower bounds for p can be deduced by the methods of this paper. However, an argument is given which makes it seem unlikely that an indefinitely high lower bound can be so deduced. (Received May 15, 1939.)

284. O. F. G. Schilling: *Regular normal extensions over complete fields.*

Let k be a complete discrete field whose residue class field is a finite field of characteristic χ. Suppose that $\chi\equiv 1 \pmod{p}$, p a prime. The author investigates regular normal extensions $K|k$, that is, extensions whose Galois groups have orders p^n. It is shown that the Galois group of the join $\sum K$ contains a well-defined everywhere dense subgroup F. This group F is a discrete group which suffices to describe the Galois theory of $K|k$. It turns out that F may be considered as the generalization of the Fuchsian groups occurring in the theory of algebraic functions. Finally, the Galois groups of certain regular extensions are interpreted in terms of factor groups taken from normal division algebras over k. (Received May 16, 1939.)

285. W. T. Scott and H. S. Wall: *A convergence theorem for continued fractions.* II.

Employing the theorem, which we recently announced, that the continued fraction (1) $1/1+a_2/1+a_3/1+\cdots$ (the a_n arbitrary complex numbers) converges if there exists a positive term convergent series $p_1+p_2+p_3+\cdots$ such that (2) $r_n|1+a_n+a_{n+1}|\geqq r_nr_{n-2}|a_n|+|a_{n+1}|$, $(n=1, 2, 3, \cdots; r_{-1}=r_0=a_1=0)$, where $r_n=p_{n+1}/p_n$, the authors find the following results: (a) Let $P(x, y)$ be any point on the curve $y=\pm(2x+1)(4x+1)^{1/2}/2x$, O the origin, and Q the point $z=x$, in the complex plane of $z=x+iy$. Then (1) converges if the a_n lie in any closed region interior to the triangle OPQ. (b) If (2) holds for real positive numbers r_n (not necessarily related to a convergent series $\sum p_n$), then the sequences of even and odd convergents of (1) have limits, finite or infinite. The limit of the sequence of even (odd) convergents is finite if actual inequality holds in (2) for an odd (even) value of n. (Received May 18, 1939.)

286. W. E. Sewell: *Continuity and degree of approximation by rational functions.*

This paper deals with the relation between the degree of approximation by rational functions with exterior poles and the continuity properties of the function approximated. (Received May 15, 1939.)

287. W. E. Sewell: *Continuity and integral approximation to an analytic function by polynomials in z and $1/z$.*

Let C be a Jordan curve in the z-plane, and let $f(z)$ be defined on C. This paper deals with the relation between the continuity properties of $f(z)$ on C and the degree of approximation to $f(z)$ by polynomials in z and $1/z$ as measured by a line integral over C. (Received May 15, 1939.)

288. W. E. Sewell: *The derivative of a polynomial on further arcs of the complex domain.*

In this paper results already established by the author (American Mathematical Monthly, vol. 44 (1937), pp. 577–578; National Mathematics Magazine, vol. 12 (1938), pp. 167–170) for certain curves and arcs of the complex domain are extended to the limaçon and the lemniscate. (Received May 15, 1939.)

289. Morgan Ward: *The ideal operators of a lattice.*

Let $\mathfrak{S}$ be a closed lattice. An operator $\theta S=\theta(S)$ on $\mathfrak{S}$ to $\mathfrak{S}$ is called an "ideal operator" if (i) $A\supset B$ implies $\theta A\supset \theta B$; (ii) $\theta A\supset A$; (iii) $\theta(\theta A)=\theta A$. The set $\mathfrak{S}'$ of values θS of θ is closed under crosscut and contains the unit I of $\mathfrak{S}$. It may be made into a closed lattice within $\mathfrak{S}$ by assigning as union, to any set of values V, the crosscut of all values containing every V. Conversely to any set $\mathfrak{S}'$ closed under crosscut and containing I there corresponds an ideal operator θ with values $\mathfrak{S}'=\theta\mathfrak{S}$. If O is any semi-ordered set, we introduce ideal operators θ in the Boolean algebra $\mathfrak{B}$ of all subsets S of O. To each element a of O corresponds the principal ideal A of all elements t of O such that $a\geqq t$ in O. Every ideal operator θ of $\mathfrak{B}$ leaving all principal ideals of O invariant gives a lattice $\mathfrak{S}'=\theta\mathfrak{B}$ in which O is isomorphically imbedded. We obtain the v-ideals or ovoid ideals of a semi-group as a special case. (Received May 5, 1939.)