ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

230. Reinhold Baer: *Rings with duals.*

The ring R' is said to be a right-dual of the ring R, if there exists a duality between the partially ordered set of all the right-ideals in R and the partially ordered set of all the right-ideals in R'. If the ring R possesses a right-identity element, and if there exists to every quotient-ring of R a right-dual possessing a left-identity element, then both the maximum and the minimum condition are satisfied by the right-ideals in R, and R meets a generalized uni-seriality requirement. A complete theory of the primary rings with right-duals may be developed, determining their structure and their dualities. (Received May 13, 1942.)

231. Nathan Jacobson: *Classes of restricted Lie algebras of characteristic p. II.*

Let $\mathfrak{A}=\Phi(x_1, \cdots, x_m)$ be an associative commutative algebra having the basis $x_1^{k_1}x_2^{k_2}\cdots x_m^{k_m}$, $0\leqq k_i\leqq p-1$ such that $x_i^p=\xi_i$ is in the field Φ of characteristic p. The restricted Lie algebras considered in this paper are the derivation algebras θ of $\mathfrak{A}$'s of the above type. The case where $\mathfrak{A}$ is a field was defined in a previous paper of the author and the case where $m=1$ and $\xi_1=1$ has been considered by Witt, by Zassenhaus and by Ho-Jui Chang. The author proves that θ is normal simple except when $p=2$ and $m=1$. The derivations of θ are all inner and if $p\geqq 5$ the automorphisms of θ have the form $D\rightarrow S^{-1}DS$ where S is an automorphism of $\mathfrak{A}$. This implies that if $p\geqq 5$, two algebras $\theta(\mathfrak{A}_1)$ and $\theta(\mathfrak{A}_2)$ are isomorphic if and only if the associative algebras $\mathfrak{A}_1$ and $\mathfrak{A}_2$ are isomorphic. (Received April 3, 1942.)

232. A. E. Ross: *On a theorem of Kloosterman.*

Kloosterman (Acta Mathematica, vol. 49 (1926)) determined conditions under which the form (1) $f=ax^2+by^2+cz^2+du^2$ would represent all large integers and pointed out that there was a finite (and actually small) number of such forms f for which his method failed to yield conclusive results. Upon close examination of these exceptional cases one notices that should one replace the original form f by a suitably chosen equivalent form $f_1=x'Ax$ (this Bulletin, abstract 45-9-369) then one can reduce the problem to the study of a related form of type (1) to which Kloosterman's sufficient conditions apply. Although the need for choosing the quotients of the upper left-hand corner principal minors of A by the order invariants as primes introduces an element of tentativeness, the actual computation is carried out without difficulty. (Received April 17, 1942.)

233. H. Schwerdtfeger: *A complete parametrization of the symplectic group.*

While all known parametrizations of the symplectic group omit certain "exceptional" elements, the parametrization derived in the present paper covers the whole group. Let ϵ be the n-rowed unit matrix and F the matrix with first row 0,0 and second row ϵ,0. A $2n$-rowed real regular matrix T is proved to be symplectic if and only if $T'FT-F=H$ is symmetric (contact condition), and satisfies the condition: $(F'-F)\cdot(H+F)$ is idempotent. To prove the sufficiency of the latter condition one has to show that for any such matrix H a symplectic T can be found with which it is associated by the said relation. By establishing a set of normal forms for H under the transformation $S'HS$ where S is a $2n$-rowed matrix for which $S'FS=F$, that is, S is the matrix with first row σ,0 and second row $0,\sigma'^{-1}$, a set of normal matrices T can be found such that RTS is the most general symplectic matrix where R is the matrix with first row ρ,0; second row $0,\rho'^{-1}$ and ρ,σ are any regular n-rowed matrices. The method has been carried through in detail for $n=2$. (Received May 1, 1942.)

Analysis

234. Einar Hille: *Notes on linear transformations.* IV. *Representation of semi-groups.*

Let $\{T_s\}$ be a semi-group of linear bounded transformations on a separable Banach space to itself, defined for $s>0$. Let T_s be weakly measurable and $\|T_s\|\leqq 1$ for $s>0$. Let $T_s(E)$ be dense in E. Put $A_h=(1/h)[T_h-I]$. For $h\to 0$, $T_hx\to x$ everywhere in E and $A_hx\to Ax$ in a dense set $D(A)$. Here A is linear, closed and ordinarily unbounded. The resolvent $R(\lambda)$ of A is the negative of the Laplace transform of T_s and is bounded for $\Re(\lambda)>0$. Conversely, T_s is expressible in terms of $R(\lambda)$ by the inversion formula for Laplace integrals which gives an interpretation of T_s as $\exp(sA)$. A further interpretation is given by $T_sx=\lim_{h\to 0}\exp[sA_h]x$, valid in $D(A)$. The method is essentially that of Stone. (Received April 6, 1942.)

235. Witold Hurewicz: *An ergodic theorem without invariant measure.*

Let E be an abstract space carrying a completely additive measure μ defined on a completely additive field Ω of subsets of E (it is assumed that a set $X\in\Omega$ with $\mu(X)=\infty$ can always be split into a countable number of sets with finite measures). Let T be a one-to-one point transformation of E on itself satisfying the conditions: (1) $X\in\Omega$ implies $T(X)\in\Omega$; (2) $T(X)\subset X\in\Omega$ implies $\mu(X-T(X))=0$ (incompressibility condition). Finally let $F(X)$ $(X\in\Omega)$ be an additive finite-valued set function, absolutely continuous with respect to the measure μ. For $X\in\Omega$, let $F_n(X)=\sum_{i=0}^{n}F(T^i(X))$, $\mu_n(X)=\sum_{i=0}^{n}\mu(T^i(X))$ $(n=0,1,2,\cdots)$. Since F_n is absolutely continuous with respect to the measure μ_n, $F_n(X)=\int_X f_n d\mu_n$, where f_n is a measurable point function defined on E. It can be shown that the sequence $\{f_n\}$ converges almost everywhere to a function ϕ, invariant with respect to T. By "almost everywhere" is meant that the points of divergence form a set M such that $\mu(T^n(M))=0$ for $n=1,2,\cdots$. If the measure μ is finite and invariant with respect to T, this theorem coincides with Birkhoff's ergodic theorem. (Received April 24, 1942.)

236. William Karush: *A sufficiency theorem for an isoperimetric problem in parametric form with general end conditions.*

The problem studied is that of minimizing an integral $I(C)=g(a)+\int_{t_1}^{t_2}f(a,y,y')dt$