The book, however, should prove useful as a reference book on the literature of the subject prior to 1938.

W. SEIDEL

Dimension Theory. By Witold Hurewicz and Henry Wallman. (Princeton Mathematical Series, no. 4.) Princeton University Press, 1941. 156 pp. $3.00.

In contrast with Karl Menger's well known *Dimensionstheorie* which could claim a considerable measure of completeness at the time of its publication (1928) the present modest volume includes "only those topics . . . which are of interest to the general worker in mathematics as well as the specialist in topology." Despite this self-imposed limitation, the authors have presented a simple and connected account of the most essential parts of dimension theory. They have treated this branch of topology—hardly surpassed in the elegance of its results—with discrimination and technical skill (it should be pointed out that the senior author is one of the outstanding builders of the theory). The result is an unusually interesting book. It must have been a pleasure to write; it was certainly a pleasure to read.

The dimension of a space is the least integer n for which every point has arbitrarily small neighborhoods whose boundaries are of dimension less than n; empty sets are of dimension -1. This well known recursive definition is due independently to Menger and Urysohn although its intuitive content goes back to Poincaré. It would be natural to expect that a theory based on such a definition would be purely point-set theoretical in character. The remarkable thing is, however, that there exist a number of equivalent definitions of dimension, each belonging to a different domain of ideas and each "right" and natural in its domain. The dimension of X can be defined, for example, as the least n such that X can be approximated arbitrarily well (in a certain sense) by polytopes of dimension not exceeding n. Or, dim X can be defined as the least n for which every continuous mapping of an arbitrary closed subset of X into an n-sphere S_n can be extended to a mapping of the whole of X into S_n. The first of these definitions brings the concept of dimension into the realm of algebraic topology (complexes, homology theory, and so on) and the second gives it a place in the theory of continuous mappings. By a skillful interplay between these two settings for dimension theory, the authors obtain a simple characterization of dimension purely in terms of homology theory. The technique which leads so smoothly to this result includes the use of cohomologies and character groups,—an indication of the thoroughly contemporary quality of the book.

The reader—specialist in topology or not—will find particularly interesting the elementary proof of Brouwer's fixed-point theorem for closed n-cells and the use of this theorem in showing that the dimension of E_n (Euclidean n-space) is n; Hurewicz's beautiful proof that every n-dimensional space can be imbedded in E_{2n+1} and the use of the technical devices in that proof in establishing the relation between dimension and the orders of coverings; the systematic use of the notion of extension (of a mapping or homomorphism); a separation theorem for homeomorphs of $(n-1)$-spheres in E_n as a corollary of a theorem about mappings; a discussion of the relation between dimension and measure; a number of remarks about spaces of infinitely many dimensions; an exposition of Čech homology theory and the dual cohomology theory; the use of cohomologies in questions about mappings and their extensions. It is understood throughout the book that "space" means "separable metric space." A short appendix contains an account of the difficulties which arise in extending the theory to more general spaces.

P. A. Smith

The Laplace Transform. By David Vernon Widder. (Princeton Mathematical Series, no. 6.) Princeton University Press, 1941. 10+406 pp. $6.00.

The theory of integral operators has developed into a major branch of analysis. It has proved to be a valuable, in fact essential, complement to the theory of differential equations, while its logical structure is more satisfactory in the sense that one deals in general with equivalences, rather than the sufficient conditions of differential equations.

As one might expect, the theory of integral operators, even in the linear case, also has complementary subdivisions. There is the well known spectral theory, which while highly effective when applicable, seems to be closely confined to functions in L_2. For other classes of functions, there have been investigations of particular operators, for example the Fourier transform.

But besides the Fourier transform, there are others which have been investigated for more than a century and it is to certain of these that the present work is devoted. However, the author also presents various applications of integral transforms (including the Fourier) of great interest and importance so that this work attains a generality which makes its subject matter of basic importance for the analyst.